

Multiple-Scattering Formalism

Beyond the Quasistatic Approximation

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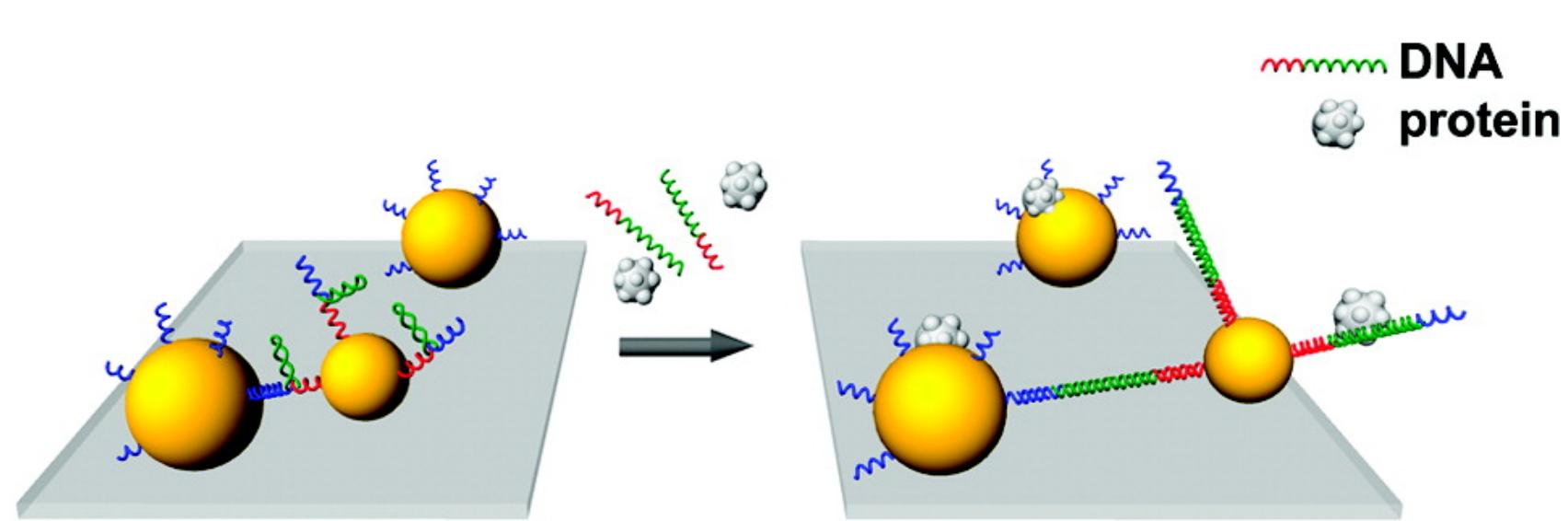
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I: WHAT APPLICATIONS OF FORMALISM?

- Modeling of optical microstructures, e.g. **3D photonic crystals**; Computation of Green's tensor, LDOS, cavity modes and Q -factors.
- Modeling of **plasmonic nanoparticles**, for instance for use in biosensing [1] and imaging and detection devices.

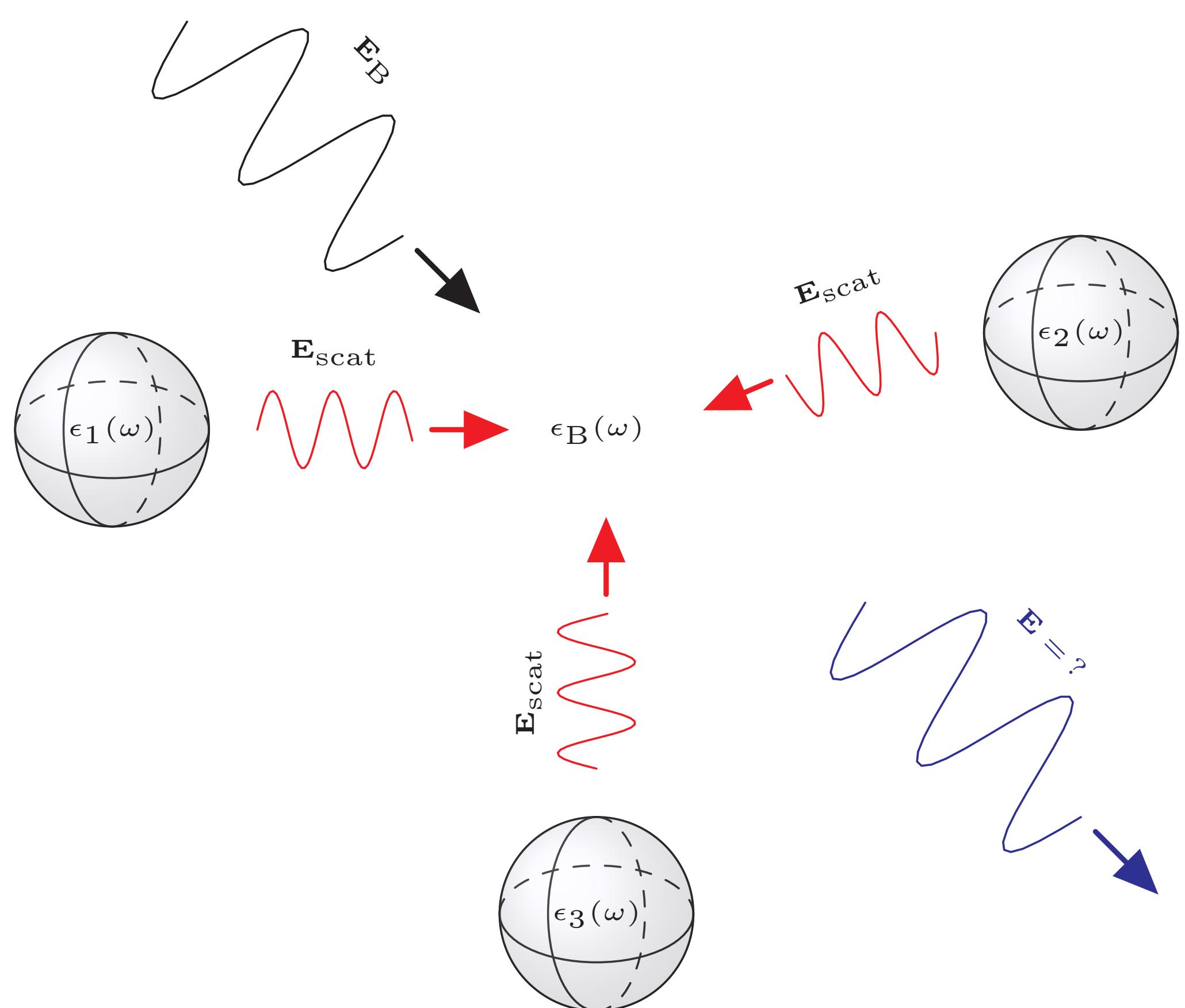


II: WHY MULTIPLE-SCATTERING FORMALISM?

- Intriguing phenomena, e.g. near-field coupling and strong field enhancements, for closely spaced metallic nanoparticles.
- Breakdown of simpler approaches**, e.g. quasistatic and dipole approximations, for closely spaced particles; Need for **modeling beyond lowest-order description**.

III: PROBLEM FORMULATION

- Rigorous determination of \mathbf{E} , resulting from scattering of \mathbf{E}_B on N spherical scatterers in homogeneous background



- Computation of derived quantities, e.g. **far-field radiation pattern**, $f(\theta, \phi)$, **extinction cross section**, C_{ext} , and **LDOS**, $\rho(\mathbf{r}; \omega)$.

IV: SOLUTION TECHNIQUE

- Lippmann-Schwinger equation

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_B(\mathbf{r}) + k_0^2 \int_{V_{\text{scat}}} \mathbf{G}_B(\mathbf{r}, \mathbf{r}') \Delta\epsilon(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}'.$$

- Implicit equation inside scatterers; Explicit outside scatterers.

- Explicit error estimate**.

- 3D generalization of [2]; Local expansion of field inside scatterers on **spherical wavefunctions**

$$\mathbf{E}(\mathbf{r}_j) = \sum_{\alpha l m} a_{j \alpha l m} \psi_{l,m}^j(\mathbf{r}_j) \mathbf{e}_\alpha, \quad \psi_{l,m}^j(\mathbf{r}_j) = K_j^l j_l(k_j r_j) Y_l^m(\theta_j, \phi_j).$$

- Lippmann-Schwinger equation as matrix equation for expansion coefficients

$$\mathbf{a} = \mathbf{M}\mathbf{a}_B + k_0^2 \mathbf{G} \Delta \epsilon \mathbf{a}.$$

- All matrix elements expressed analytically.

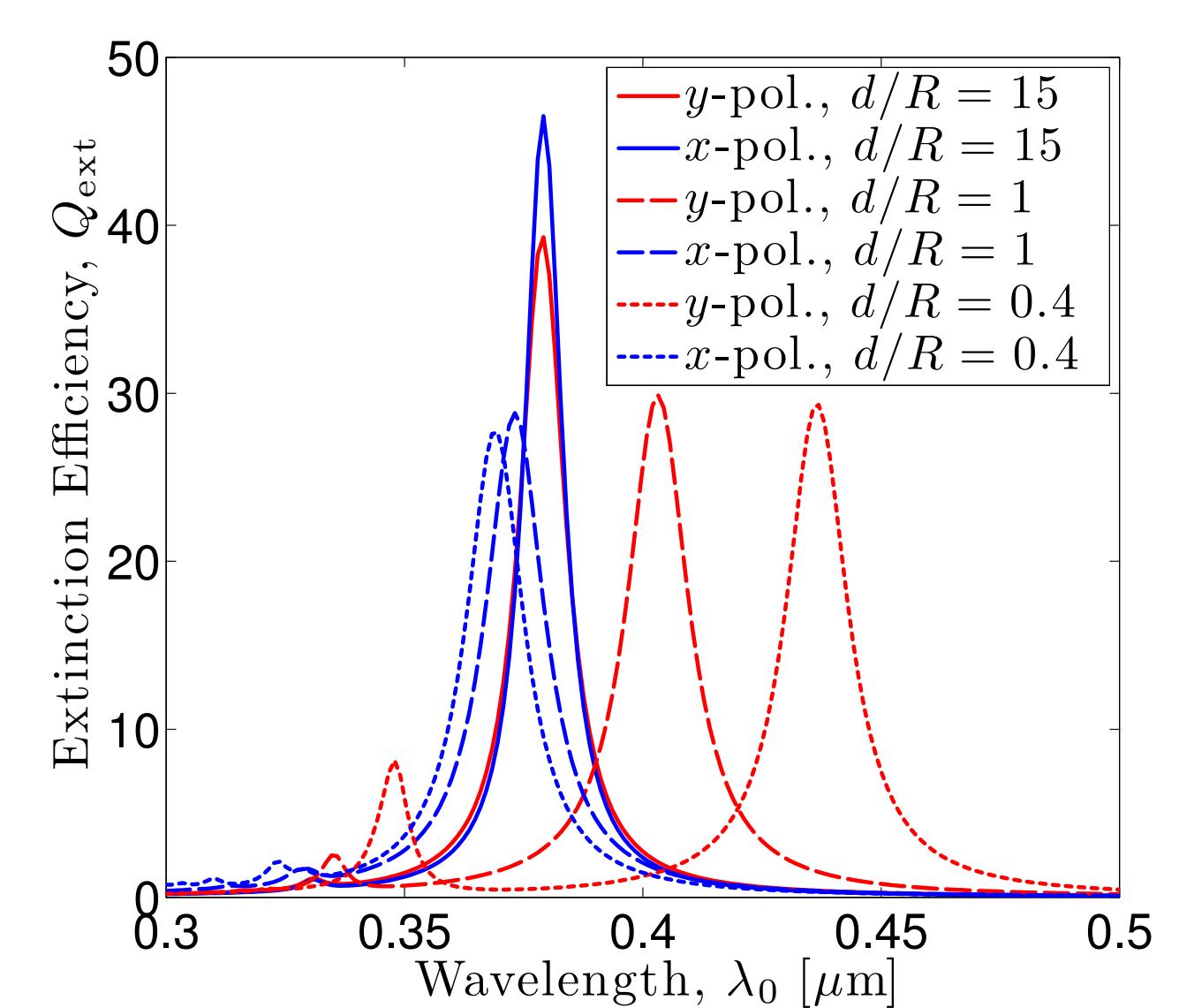
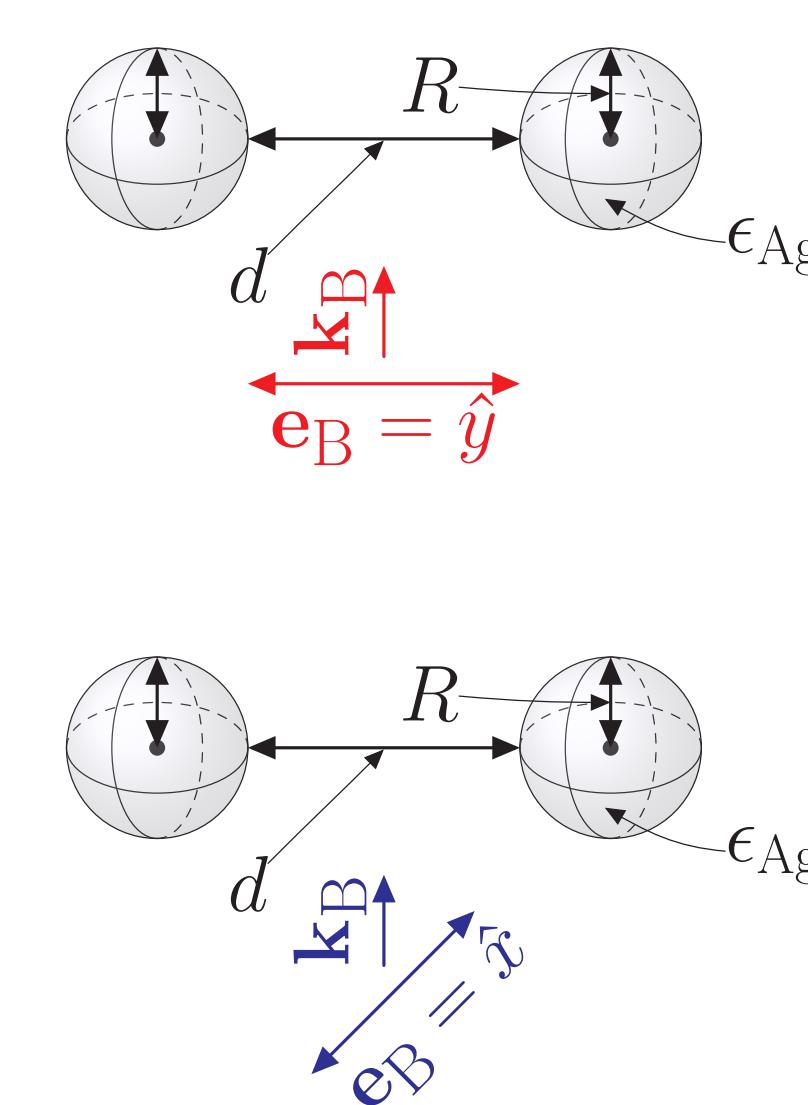
REFERENCES

[1] J. I. L. Chen, Y. Chen, and D. S. Ginger, *J. Am. Chem. Soc.* **132**, 9600 (2010).

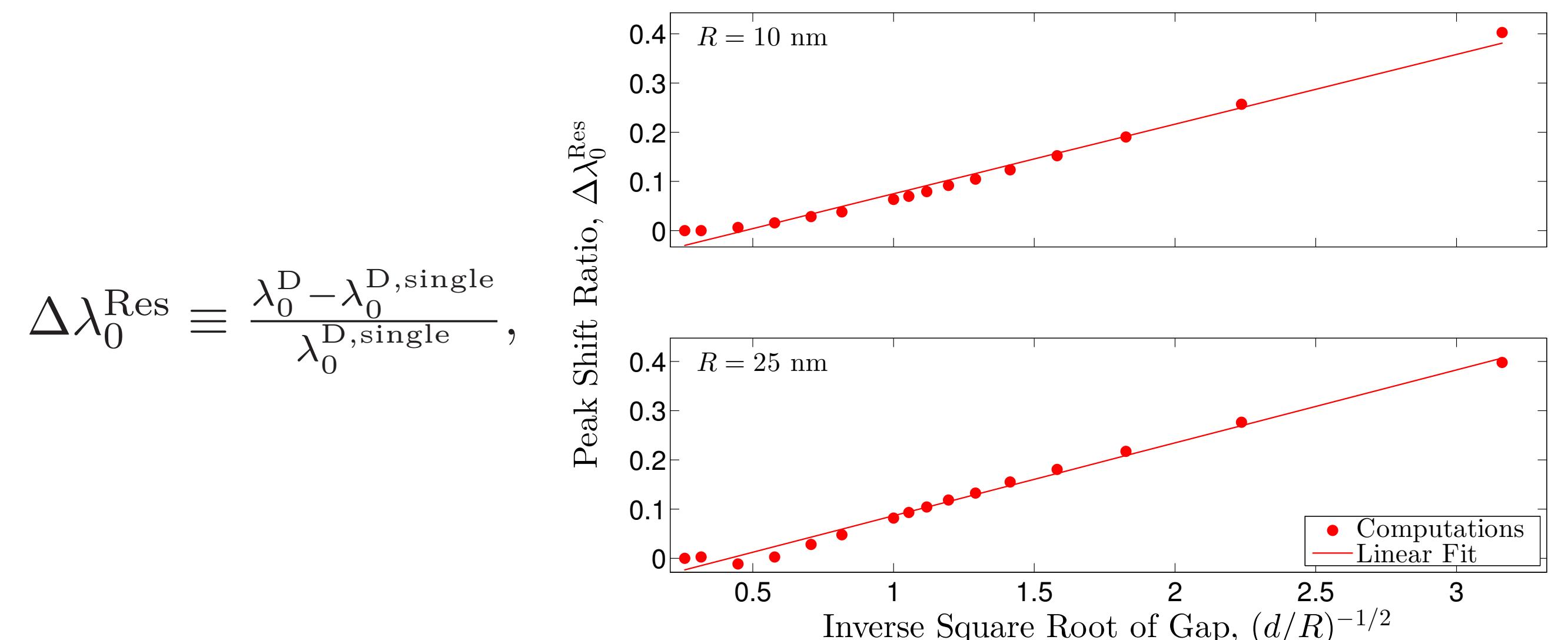
[2] P. T. Kristensen, P. Lodahl, and J. Mørk, *J. Opt. Soc. Am. B* **27**, 228 (2010).

V: PLASMONIC DIMER

- Plane wave excitation; Parallel (\hat{y}) and perpendicular (\hat{x}) polarizations



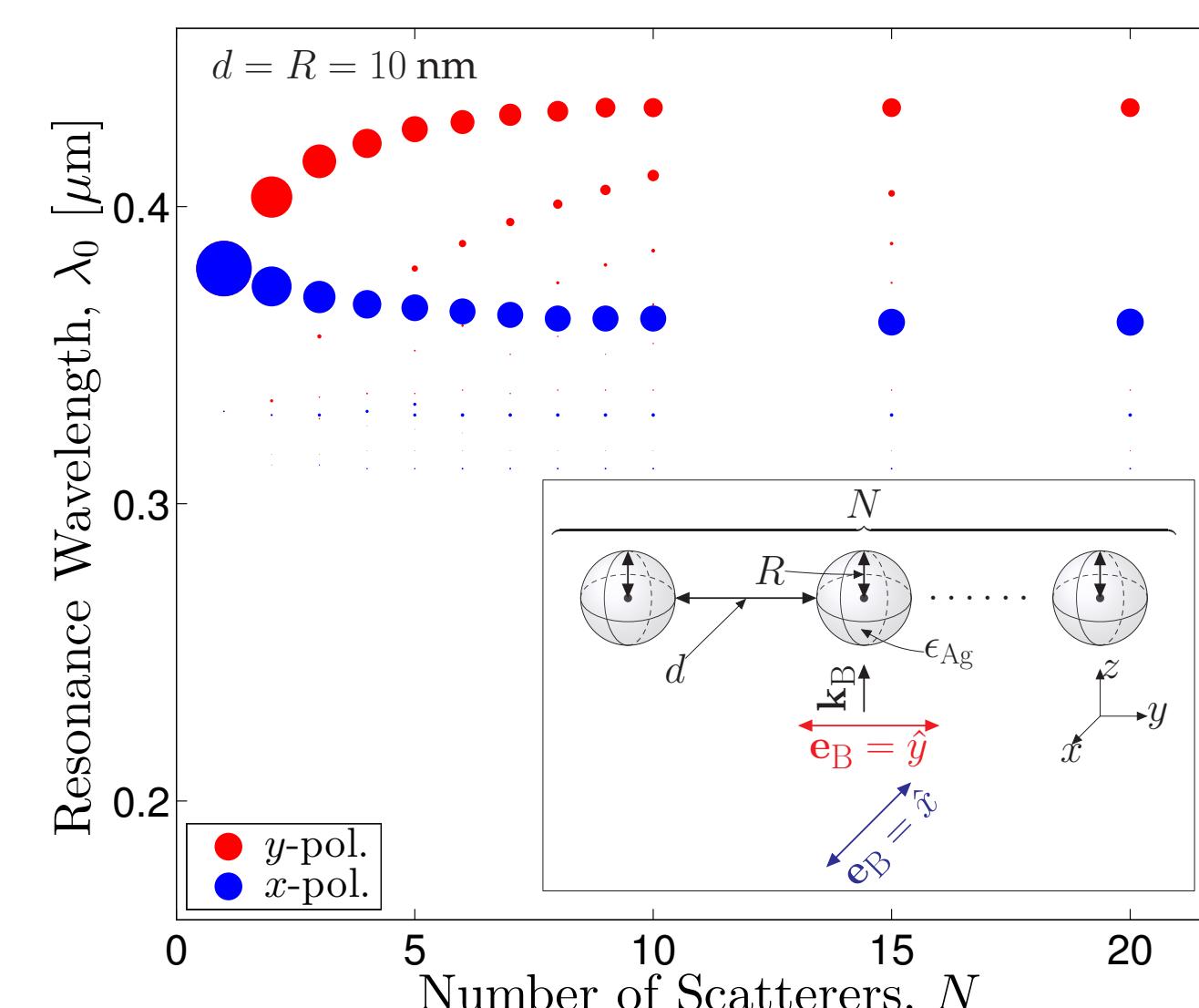
- Distinct redshift for decreasing d/R for parallel polarization.**
- Correlate peak wavelength with d/R through **peak shift ratio**



- $\Delta\lambda_0^{\text{Res}} \propto \exp(-(d/R)/\eta)$ [3]; $\Delta\lambda_0^{\text{Res}} \propto 1/(d/R)$ [4]; Not agreement.

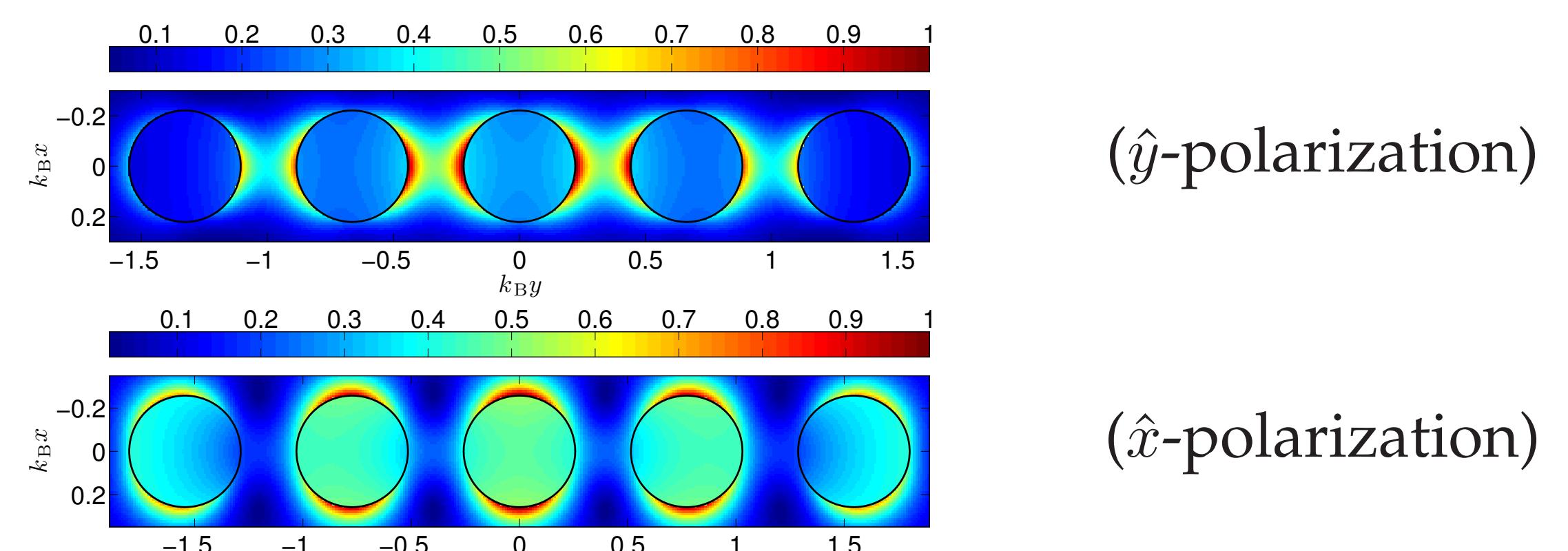
VI: PLASMONIC CHAIN

- Resonance wavelengths as function of number of scatterers



- $\Delta\lambda_0^{\text{Res}} = \Delta\lambda_0^{\text{Asymp}} \exp(-\eta_N/(N-1))$, $\eta_N \approx 2$; Agreement with [4].

- Two polarizations; **Two types of localized surface plasmons**



VII: CONCLUSIONS

- Multiple-scattering formalism based on Green's tensor; Easy and intuitive access to, e.g., extinction scattering spectra, LDOS and cavity modes/quasinormal modes.
- Plasmonic dimer: Correlation between spectrum and particle spacing (\hat{y} -pol.).
- Plasmonic chain: Finite interaction length along chain (\hat{y} -pol.).

[3] P. K. Jain, W. Huang, and M. A. El-Sayed, *Nano Lett.* **7**, 2080 (2007).

[4] N. Harris, M. D. Arnold, M. G. Blaber, and M. J. Ford, *J. Phys. Chem. C* **113**, 2784 (2009).