Modeling of cavities using the analytic modal method and an open geometry formalism

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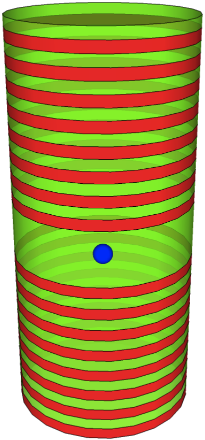
Outline

- Introduction.
- Modeling schemes and boundary conditions.
- Open geometry formalism.
- Results.

Modeling of Electric Field and Purcell Factor

- Modeling of electric field and Purcell factor in microstructured, dielectric cavity, due to dipolar source.
- Purcell factor an important figure of merit for cavity, $F_p = \frac{Q}{V}\frac{3\lambda^3}{4\pi^2 n^3}$.
- Assuming unity quantum efficiency, $F_p = P/P_0$, where P and P_0 are the powers radiated in the cavity and in bulk, respectively.[1]
- The power is proportional to the real part of the electric field, $P \propto \mathrm{Re}(E)$.

Accurate modeling of electric field and Purcell factor desired.



[1] L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University Press, 2006), 1st ed.

Modeling Schemes

Techniques for modeling electric field:

- FDTD, FEM. Memory requirements.
- Green's function/tensor.
- Modal expansion methods:
  - Plane wave expansions.
  - Eigenmode expansions. Physically intuitive.
    - Metallic boundary conditions, periodic boundary conditions.
    - PML boundary conditions.
    - No (exterior) boundary conditions. Open geometry.

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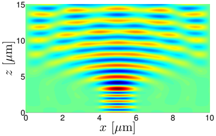

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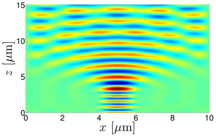

Widely used. Artificial, absorbing boundaries. Dependence on arbitrary parameters.

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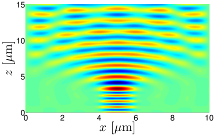

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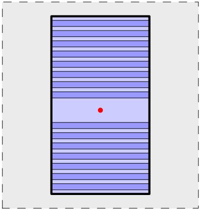
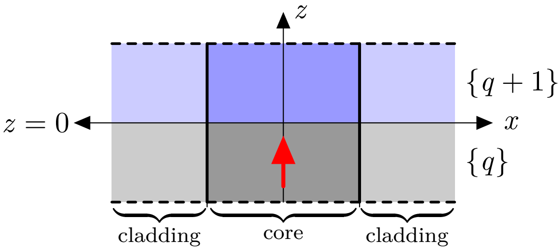
Emulation of physical system. Technique that we have pursued.

Open Geometry: Outline of Method





- Expand the field in each layer on analytical eigenmodes:

$$E_q(\mathbf{r}) \sim \sum_j c_j U_j^q(\mathbf{r}) + \int_{-\infty}^{\infty} c(\rho)\psi^q(\mathbf{r}, \rho)\, \mathrm{d}\rho. \tag{1}$$

- Consider generic interface and compute reflection and transmission matrices.
- Use scattering matrix formalism to express field in the full structure.[2]

[2] L. Li, J. Opt. Soc. Am. A **13**, 1024-1035 (1996).

Open Geometry: Matrix Equation

- Impose boundary conditions of electric field at interface. Obtain Lippmann-Schwinger equation for interface field, $\Phi(x)$:

$$E(x, z=0) \equiv \Phi(x) = \Phi_0(x) + \int_{-\infty}^{\infty} K(x, x')\Phi(x')\,\mathrm{d}x'. \quad (2)$$

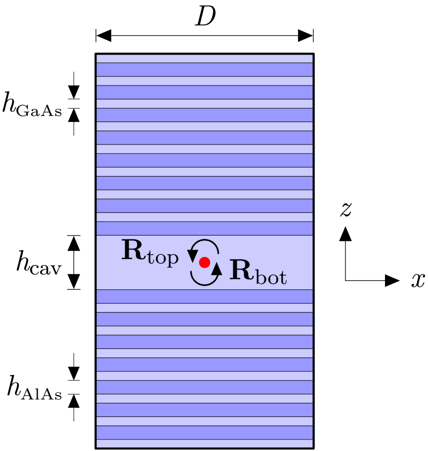
- The field expansions coefficients, c_j and $c(\rho)$ depend on $\Phi(x)$.
- Expand $\Phi(x)$ on eigenmodes, $\Phi(x) = \sum_j c_j U_j^q(x)$ $+ \int_{-\infty}^{\infty} c(\rho)\psi^q(x,\rho)\,\mathrm{d}\rho$, and convert implicit integral equation into matrix equation through a discretization:[3]

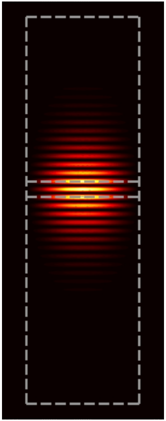
$$\mathbf{c} = \mathbf{c}^0 + \mathbf{K}\mathbf{c} \Leftrightarrow \mathbf{c} = (\mathbf{I} - \mathbf{K})^{-1}\mathbf{c}^0. \quad (3)$$

- Compute reflection and transmission matrices, $\mathbf{R}$ and $\mathbf{T}$, to be used in scattering matrix formalism.

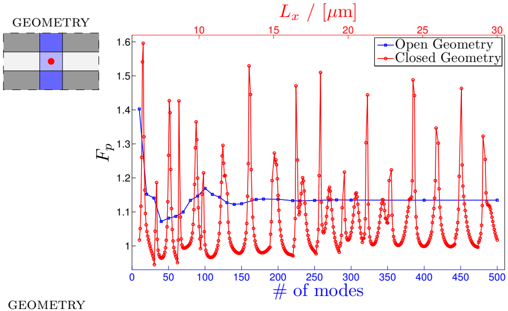
[3] P. T. Kristensen, P. Lodahl, and J. Mørk, J. Opt. Soc. Am. B **27**, 228-237 (2010).

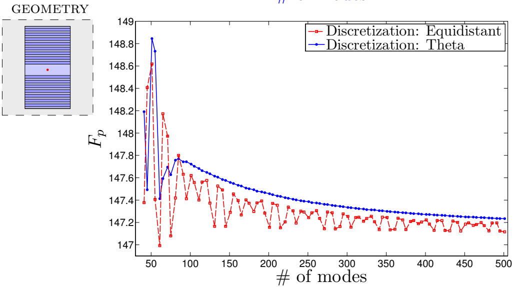
Rectangular, 2D Micropillar: Proof of Concept





Numerical Results: Purcell Factor





Purcell factor converges using the open geometry formalism.

Conclusion and Outlook

- Proof of concept: Open geometry method allows for a convergent computation of the Purcell factor.
- Formalism, in principle, applicable where analytical eigenmodes are available (e.g. cylindrical or elliptical micropillars).
- Extension to more realistic 2D and 3D structures.

Questions?