Optical Simulations in an Open Geometry Using the Eigenmode Expansion Technique

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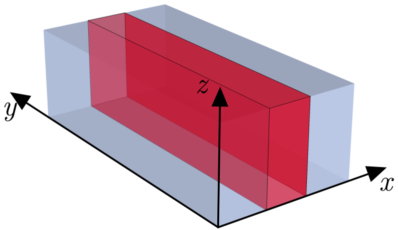
Introduction

Outline of Thesis

- Modeling of the $\mathbf{E}$-field in slab structures illuminated by TE-waves.



- Two approaches: The closed and the open geometry.
- Field profiles and the normalized spontaneous emission rate used to assess the two approaches.

Wave Equations

Maxwell's Equations → Scalar Helmholtz Equation

- Starting point: Maxwell's equations. 3D-problem.
- Reduction to a 1D-problem:
  - No free charges/currents and constant material parameters give Helmholtz equation:

$$\nabla^2 \mathbf{E} + n^2 k_0^2 \mathbf{E} = 0.$$

  - Slab structures: $\mathbf{E}(x, y, z) \equiv \mathbf{E}(x, z)$.
  - TE-waves polarized along y:

$$\nabla^2 E_y(x, z) + n^2 k_0^2 E_y(x, z) = 0.$$

  - Separation of variables, $E_y(x, z) = e_x(x) e_z(z)$, yields solutions:

$$e_x(x) \propto \exp(\pm i\kappa x), \qquad e_z(z) \propto \exp(\pm i\beta z).$$

The Closed Geometry Approach

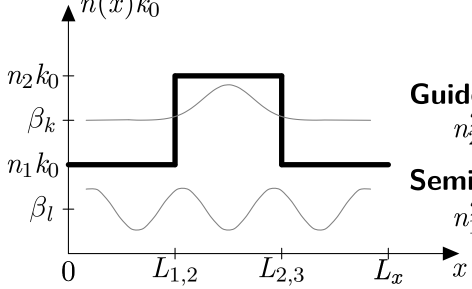
Boundary Conditions

- Main characteristic: Finite x-width. Metallic boundaries.
- Boundary conditions (BCs):
  - Inner BCs:
    - Continuity and differentiability of field in all points.
  - Outer BCs:
    - Vanishing field at $x = 0$ and $x = L_x$: $e_x(0) = e_x(L_x) = 0$.
    - z-BCs: Illumination.
- The x-BCs are handled by the **semi-analytical approach** and ensure a discrete, complete and orthonormal set of eigenmodes.
- The z-BCs are handled by the **scattering matrix formalism**.

The Closed Geometry Approach

Eigenmodes

- Two types of eigenmodes:



Guided modes:

$$n_2^2k_0^2 > \beta_k^2 > n_1^2k_0^2.$$

Semi-radiating modes:

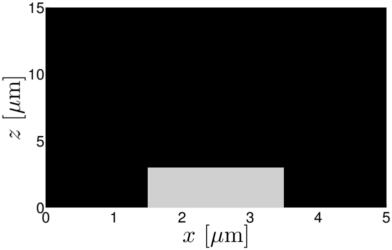
$$n_1^2k_0^2 > \beta_l^2 > -\infty.$$

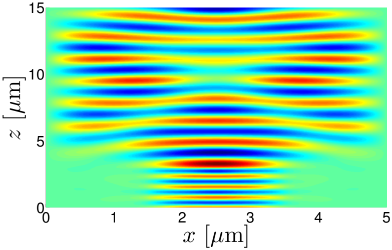
- Eigenmode expansion: $E_{\text{closed}} \sim \sum_j e_{x,j}e_{z,j}$.

The Closed Geometry Approach

Field Profile: Abruptly Terminated Waveguide

Abruptly terminated waveguide illuminated by fundamental mode.





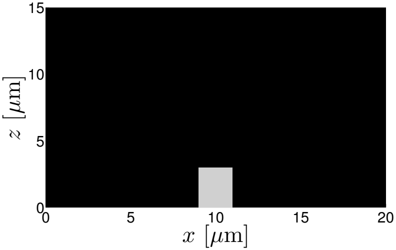
Small solution domain.

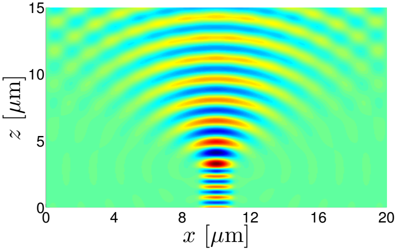
Parasitic reflections!

The Closed Geometry Approach

Field Profile: Abruptly Terminated Waveguide

Abruptly terminated waveguide illuminated by fundamental mode.



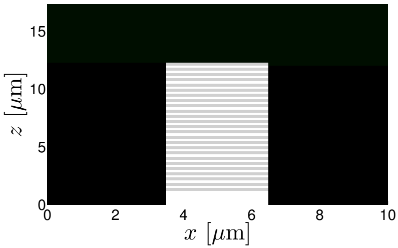


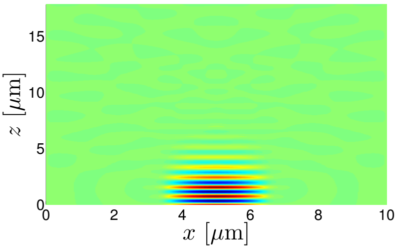
Large solution domain.

The Closed Geometry Approach

Field Profile: Bragg Grating

Bragg grating illuminated by fundamental mode.





Grating dimensions tuned to reflection at $\lambda = 1.55\,\mu\mathrm{m}$.

The Closed Geometry Approach

Spontaneous Emission Rate: Theory

- The normalized spontaneous emission rate (SER):

$$\alpha \equiv \frac{\gamma}{\gamma_0}.$$

- Equivalent quantity:

$$\alpha = \frac{P}{P_0}.$$

- The power emitted from a dipole is proportional to the field due to the dipole, evaluated in the position of the dipole:

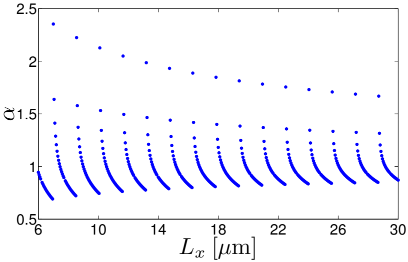
$$P = -\frac{1}{2}\mathrm{Re}\big(E(x_c, z_c)\big).$$

- The normalized SER should, in principle, be calculable by increasing L_x until convergence.

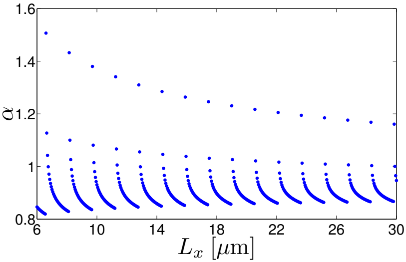
The Closed Geometry Approach

Spontaneous Emission Rate: Results

Single-layers with uniformity along z.



Vacuum layer



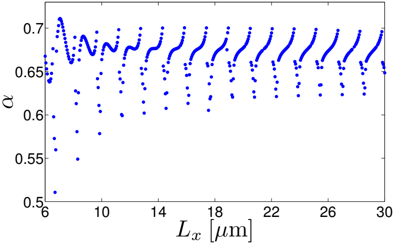
Waveguide layer

No convergence!

The Closed Geometry Approach

Spontaneous Emission Rate: Results

Three-layer structure. Waveguide in vacuum.



Extension to multi-layered structures straightforward.

The Open Geometry Approach

Presentation

- No outer x-BCs: $\mathbf{E}(x,z)$ is nowhere forced to vanish.
- More distinct mode types:
  - Guided modes: $U_j(x)$. Discrete set, finite number.
  - Radiation modes: $\phi_l(x,s)$ or $\psi_m(x,\rho)$. Continuum.
- The eigenmode expansion:

$$E_{\text{closed}} \sim \sum_j e_{x,j} \longrightarrow E_{\text{open}} \sim \sum_j U_j + \sum_{m=1}^{2} \int_0^\infty \psi_m(\rho)\,\mathrm{d}\rho.$$

Different treatment of radiation modes.

The Open Geometry Approach

Spontaneous Emission Rate in Single-Layer Structure: Theory

- As in closed geometry: $P = -\frac{1}{2}\mathrm{Re}\big(E(x_c, z_c)\big)$. Field due to dipole required.
- The power emitted in a uniform layer has an analytical solution:

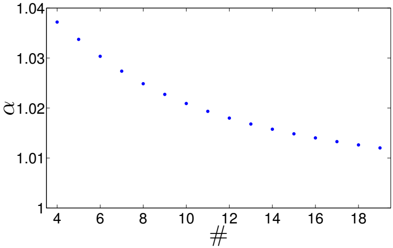
$$P_0 = \frac{1}{8}.$$

- P_0 is the reference power in all SER-computations.
- Power in waveguide single-layer structures require approximation of improper integrals, cf. eigenmode expansion.

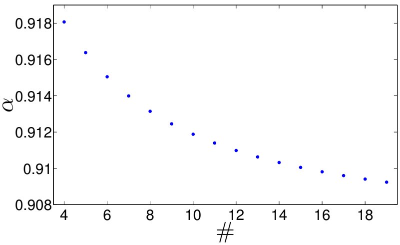
The Open Geometry Approach

Spontaneous Emission Rate in Single-Layer Structure: Results

θ-sampling of integrals: Rapidly converging results.



Vacuum layer



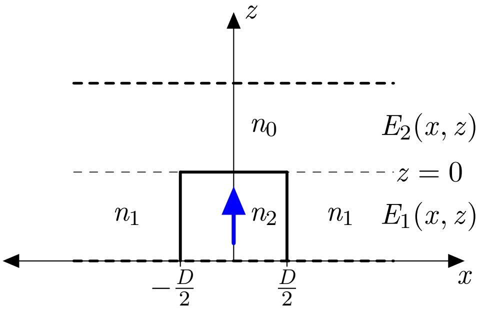
Waveguide layer

Convergence: Relative deviation of 1% with 24 (vacuum) and 7 (waveguide) sampling points.

The Open Geometry Approach

Two Layers: Abruptly Terminated Waveguide

Reflection and transmission between waveguide and uniform layer.



The Open Geometry Approach

Reflection and Transmission Coefficients

The fields in each layer, $E_1(x,z)$ and $E_2(x,z)$, given by the eigenmode expansion:

$$\begin{aligned} E_1(x,z) &= U_1(x)\left[\exp(-i\beta_1 z) + R_1 \exp(i\beta_1 z)\right] \\ &\quad + \sum_{m=1}^{2} \int_0^\infty R(\rho)\psi_m(x,\rho)\exp(i\beta(\rho)z)\,\mathrm{d}\rho, \\ E_2(x,z) &= \sum_{l=1}^{2} \int_0^\infty T(s)\phi_l(x,s)\exp(-i\gamma(s)z)\,\mathrm{d}s. \end{aligned}$$

Reflection and transmission coefficients, R_1, $R(\rho)$, and $T(s)$, to be determined.

The Open Geometry Approach

Aperture Field

- Continuity and differentiability of the field at $z = 0$ and use of orthonormality and completeness relations gives a Fredholm Equation of the Second Kind for the aperture field, $\Phi(x)$:

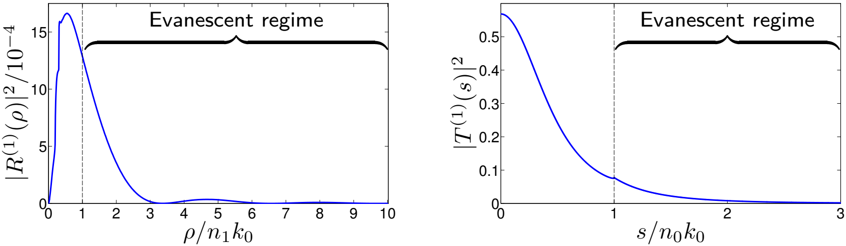
$$\Phi(x) = \Phi_0(x) + \lambda \int_{-\infty}^{\infty} \Phi(x') K(x, x') \, \mathrm{d}x'.$$

- R_1, $R(\rho)$, and $T(s)$: Functions of $\Phi(x)$.
- Assuming uniform convergence, an approximate solution to the integral equation is a truncated Liouville-Neumann series:

$$\Phi_N(x) = \Phi_0(x) + \lambda^N \sum_{j=1}^{N} C_j(x).$$

The Open Geometry Approach

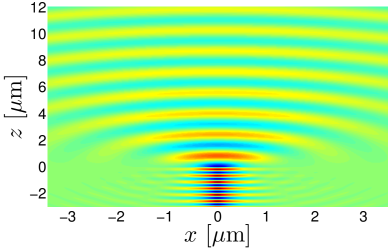
First Order Reflection and Transmission Coefficients



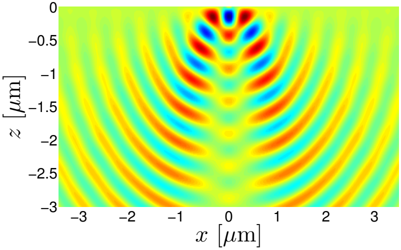
- Evanescent regime: Exponentially decaying along the propagation direction. Near-field.
- Propagating regime: Far-field.

The Open Geometry Approach

First Order Field Profile



Total field



Radiation modes field

- No parasitic reflections.
- Radiation modes in the waveguide layer: Near-field effect.

Conclusion

- The closed geometry approach:
  - Illustrative field profiles, but parasitic reflections.
  - Fluctuating SER results: Relative deviation of 8%-26%. Primarily due to semi-radiating modes.
  - Versatile tool for modeling of arbitrary structures.
- The open geometry approach:
  - Rapidly converging single-layer SER-results using θ-sampling. Relative deviation of 1% with 24 and 7 samples.
  - Abruptly terminated waveguide:
    - Incident guided mode: Field profile without parasitic reflections.
    - Incident radiation mode: Further work needed.
  - SER calculations in arbitrary open geometry possible when treatment of incident radiation mode is complete.

Open geometry: More accurate results.