A Bloch Mode Expansion Approach for Analyzing Quasi-Normal Modes in Open Nanophotonic Structures

Jakob Rosenkrantz de Lasson, Philip Trøst Kristensen,
Jesper Mørk and Niels Gregersen

Technical University of Denmark

www.nanophotonics.dk

SPIE Photonics Europe, April 15 2014

DTU Fotonik
Department of Photonics Engineering

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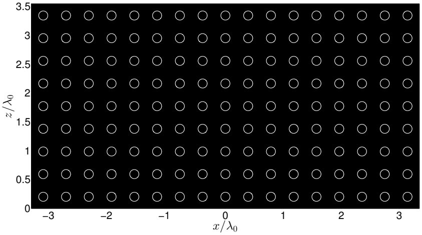
Scattering on Open Photonic Resonator – Modes?



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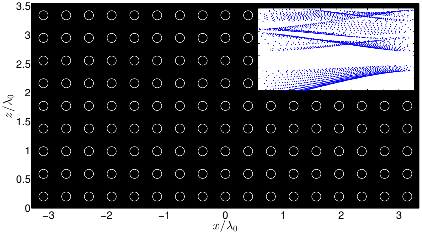

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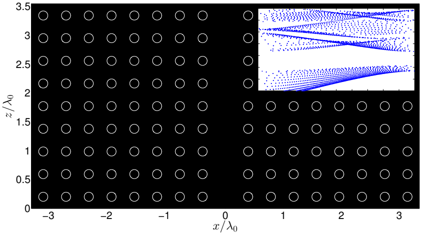

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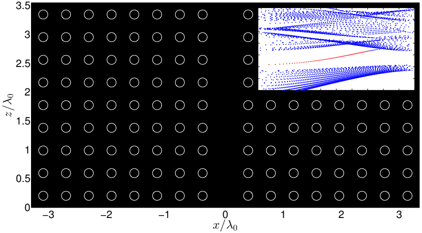

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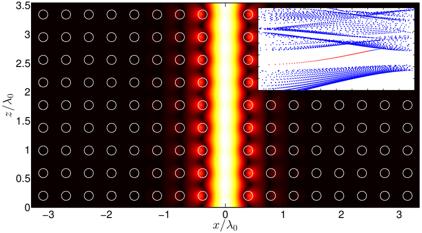

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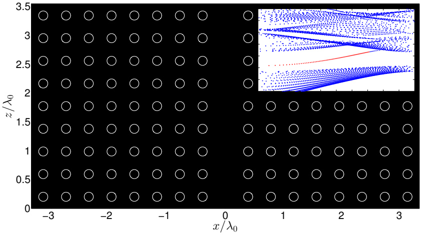

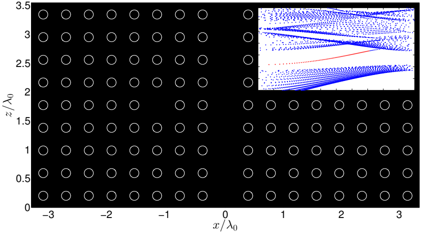
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x/λ_0

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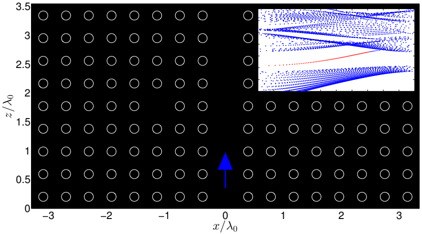

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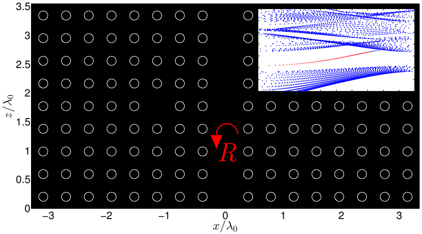

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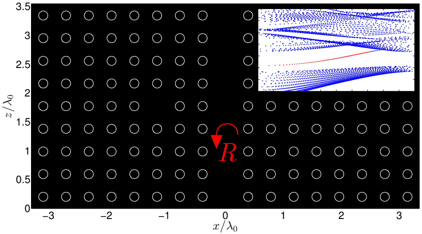

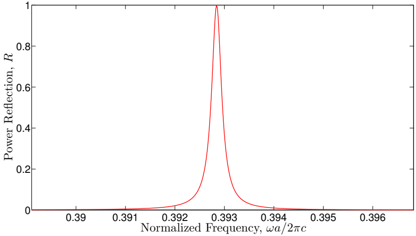

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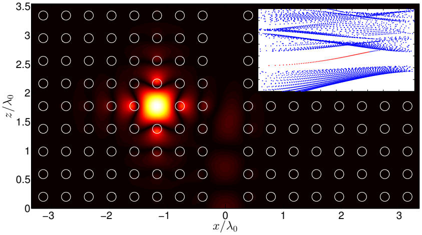

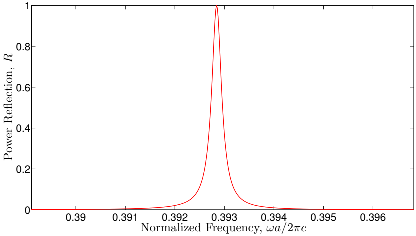

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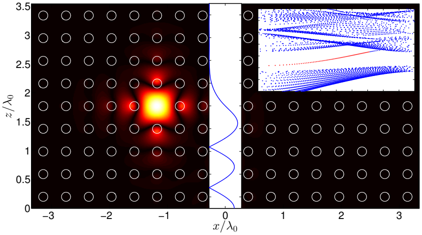

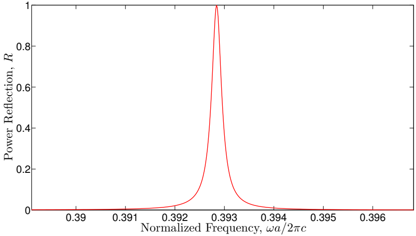

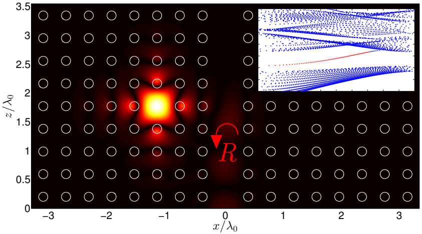

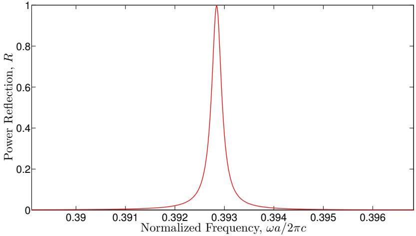

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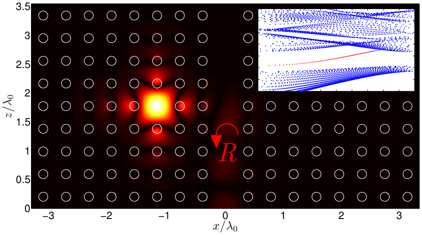

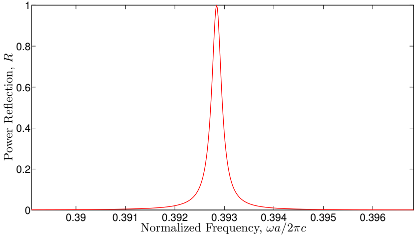

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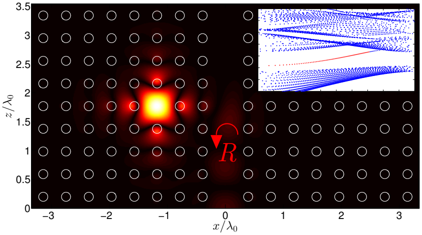

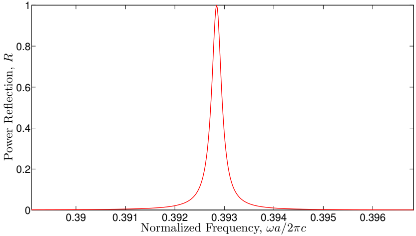

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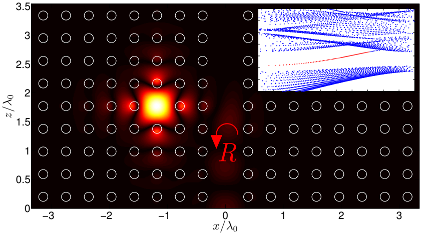

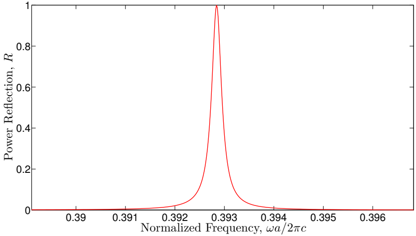

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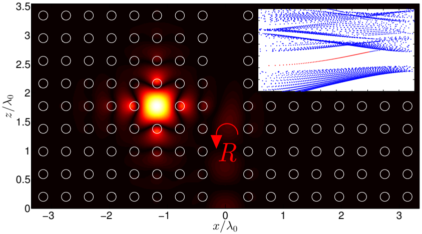

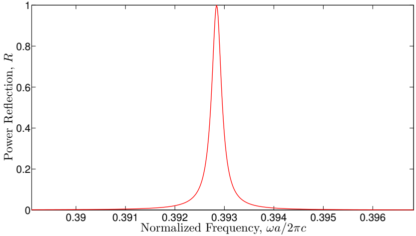

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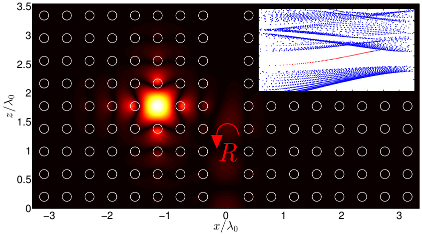
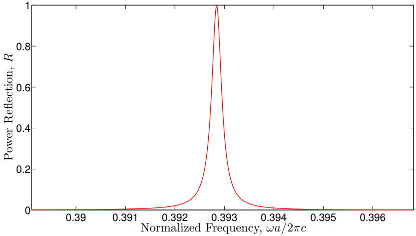
Implicit assumption of the existence of a resonator mode





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Implicit assumption of the existence of a resonator mode

. . . How to formalize the description of these modes?

Outline

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- Definition of and previous work on quasi-normal modes

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- Bloch mode expansion technique for calculating quasi-normal modes

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- Quasi-normal modes in photonic crystal cavities side-coupled to W1 waveguide

Quasi-Normal Modes: Definition

Definition: Time-harmonic solutions $\mathbf{E}(\mathbf{r};t) = \mathbf{E}(\mathbf{r};\omega)\exp(-\mathrm{i}\omega t)$ of $\nabla \times \nabla \times \mathbf{E} = \left(\frac{\omega}{c}\right)^2 \epsilon\mathbf{E}$

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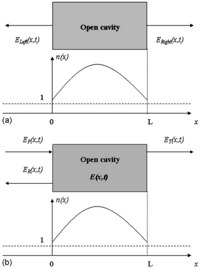
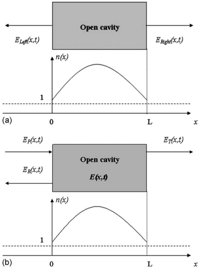
Explicit description of time-decaying resonator mode.

Quasi-Normal Modes: Selection of Previous Work



[A. Settimi *et al.*, J. Opt. Soc. Am. B **26**, 876-891 (2009)]



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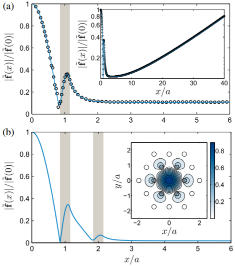

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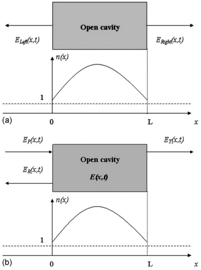

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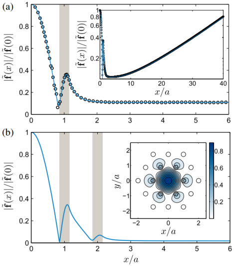

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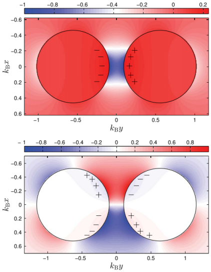

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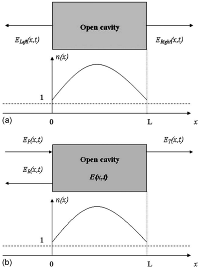

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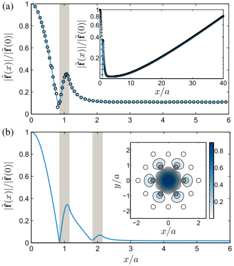

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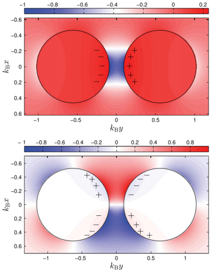

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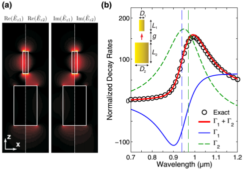
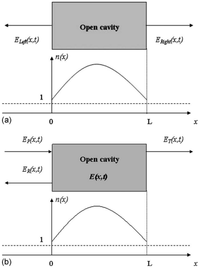


[C. Sauvan *et al.*, Phys. Rev. Lett. **110**, 237401 (2013)]



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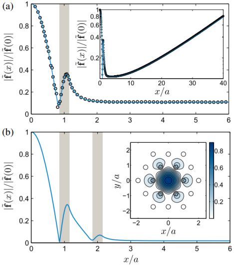

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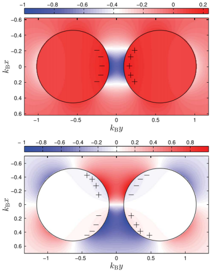

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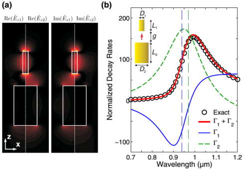

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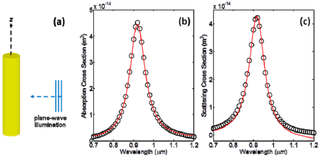
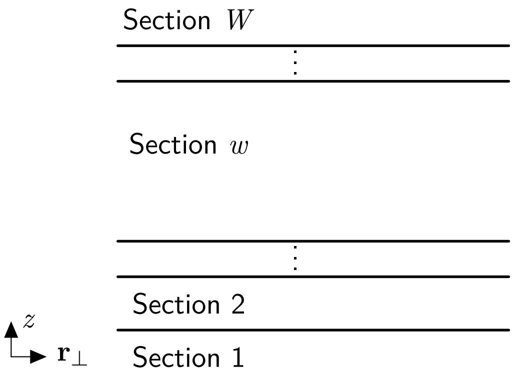


[Q. Bai *et al.*, Opt. Express **21**, 27371-27382 (2013)]


Section W
Section w
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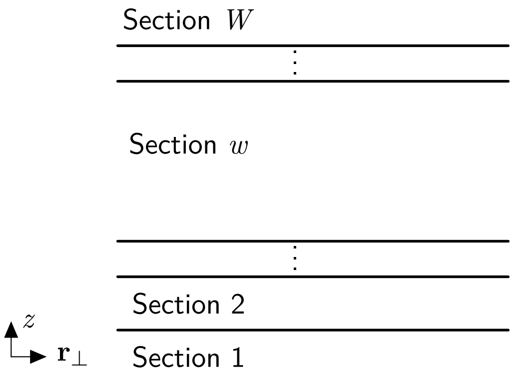
z
$\mathbf{r}_\perp$

Bloch Mode Expansions: Calculating Quasi-Normal Modes

$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z),$$



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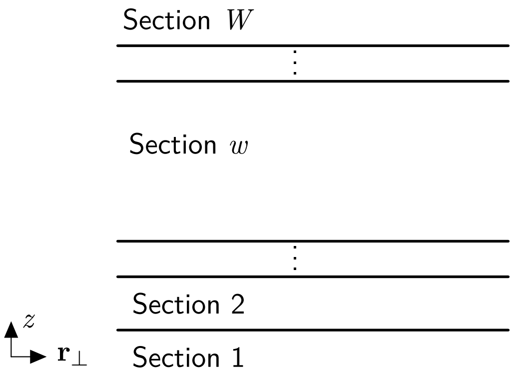

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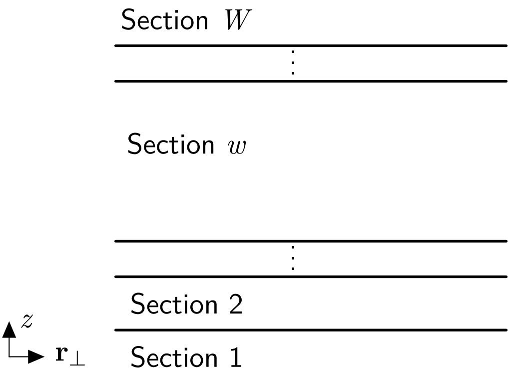

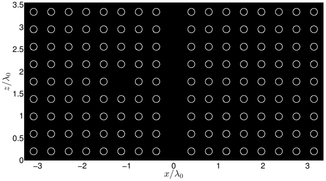

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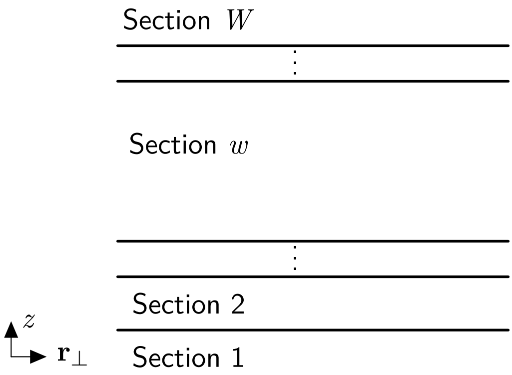

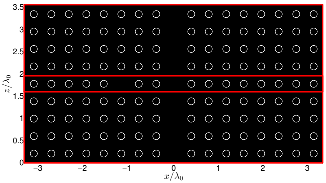

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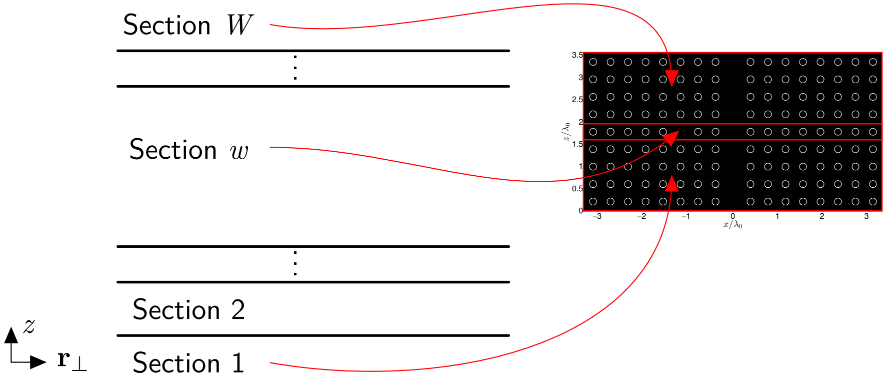

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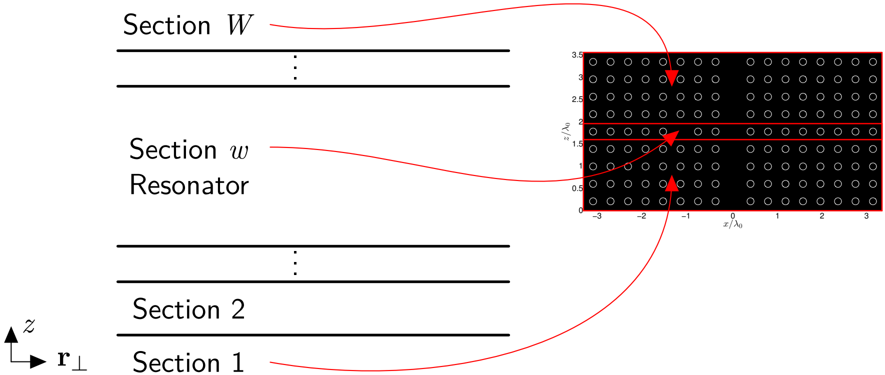

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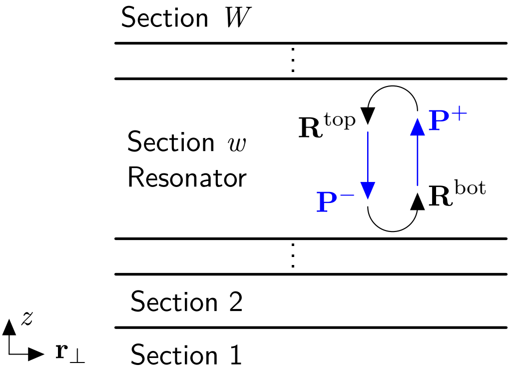

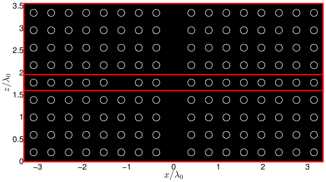

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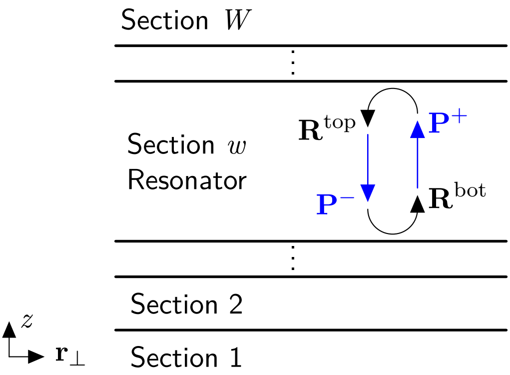
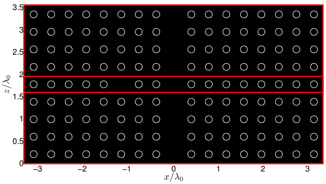


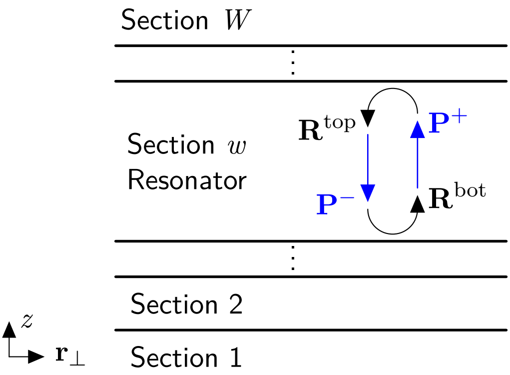


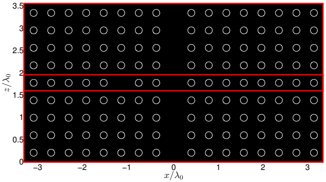
Quasi-normal mode condition:

Resonator roundtrip matrix $\mathbf{M}(\omega) \equiv \mathbf{R}^{\mathrm{bot}}\mathbf{P}^{-}\mathbf{R}^{\mathrm{top}}\mathbf{P}^{+}$ satisfying

Bloch Mode Expansions: Calculating Quasi-Normal Modes

$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z), \qquad \mathbf{e}_j^w(\mathbf{r}_\perp, z+a) = \exp\left(\mathrm{i}k_j^w a\right) \mathbf{e}_j^w(\mathbf{r}_\perp, z)$$



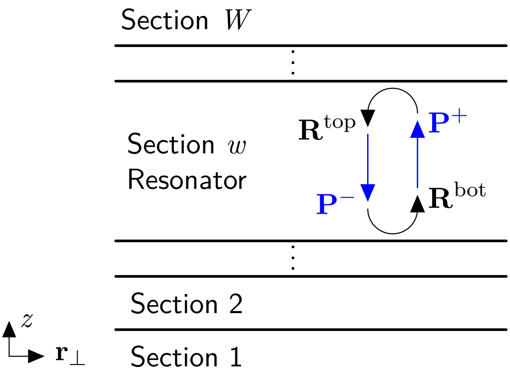


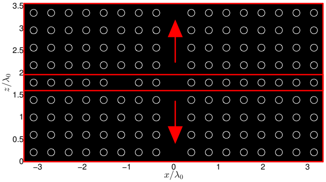
Quasi-normal mode condition:

Resonator roundtrip matrix $\mathbf{M}(\omega) \equiv \mathbf{R}^{\mathrm{bot}}\mathbf{P}^{-}\mathbf{R}^{\mathrm{top}}\mathbf{P}^{+}$ satisfying

$$\mathbf{M}(\omega)\mathbf{c}^w = \lambda_{\mathrm{R}}\mathbf{c}^w, \quad \lambda_{\mathrm{R}} = 1$$

$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z), \qquad \mathbf{e}_j^w(\mathbf{r}_\perp, z+a) = \exp\left(\mathrm{i}k_j^w a\right) \mathbf{e}_j^w(\mathbf{r}_\perp, z)$$





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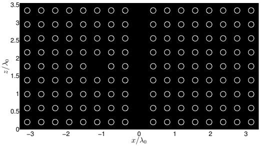
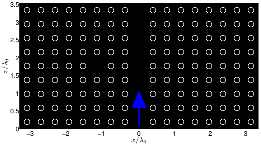
Quasi-Normal Modes in Side-Coupled PhC Cavities




3.5
3
2.5
2
1.5
1
0.5
0
z/λ_0
−3
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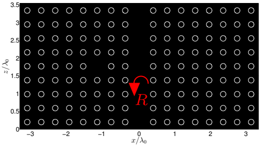
1
2
3
x/λ_0


3.5
3
2.5
2
1.5
1
0.5
0
z/λ_0
R
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−2
−1
0
1
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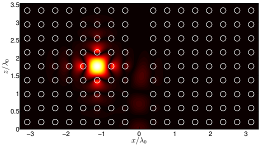
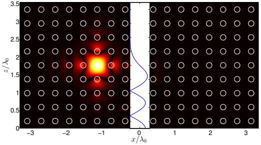
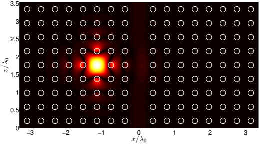
x/λ_0

Quasi-Normal Modes in Side-Coupled PhC Cavities



Quasi-Normal Modes in Side-Coupled PhC Cavities





z/λ_0
3.5
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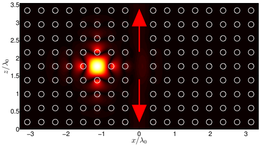
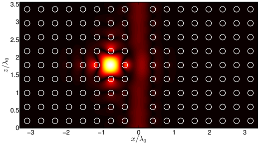
x/λ_0

Quasi-Normal Modes in Side-Coupled PhC Cavities




z/λ_0
x/λ_0
3.5
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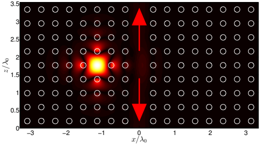
z/λ_0
x/λ_0
3.5
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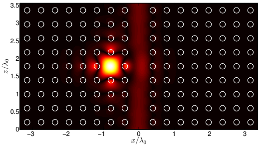
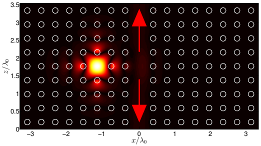
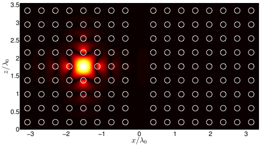
Quasi-Normal Modes in Side-Coupled PhC Cavities

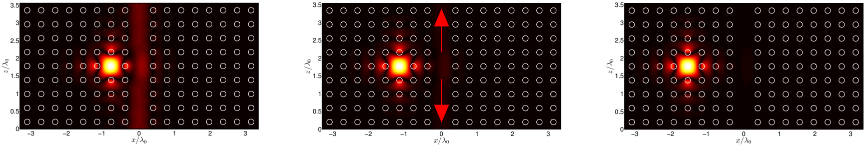






Quasi-Normal Modes in Side-Coupled PhC Cavities

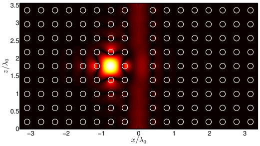
$$Q = \omega_{\mathrm{R}}/\gamma$$

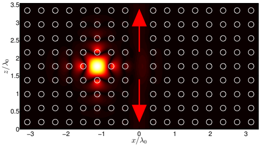


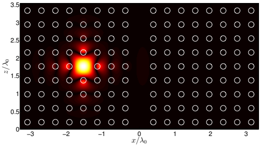
Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \omega_{\mathrm{R}}/\gamma$$

$$Q_3 = 1.6 \cdot 10^3$$



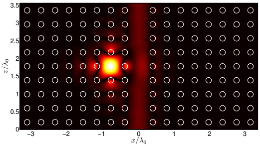


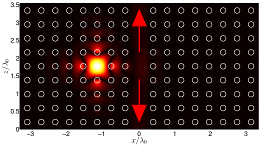


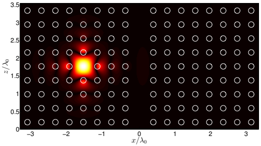
Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \omega_{\mathrm{R}}/\gamma$$

$Q_2 = 1.5 \cdot 10^2$ $\qquad$ $Q_3 = 1.6 \cdot 10^3$







Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \omega_{\mathrm{R}}/\gamma$$

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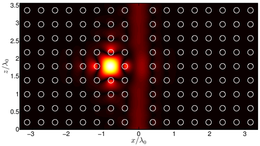
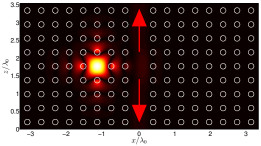
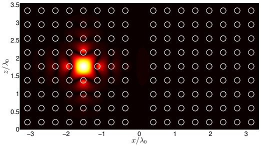


$Q_3 = 1.6 \cdot 10^3$



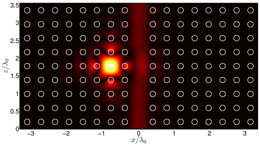
$Q_4 = 2.0 \cdot 10^4$



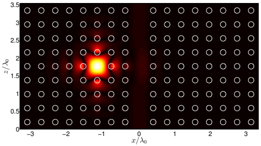
Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \omega_{\mathrm{R}}/\gamma$$

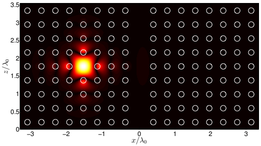
$Q_2 = 1.5 \cdot 10^2$



$Q_3 = 1.6 \cdot 10^3$



$Q_4 = 2.0 \cdot 10^4$



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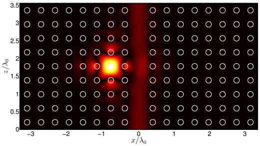
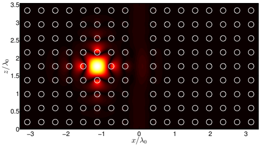
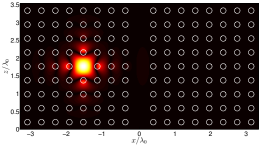
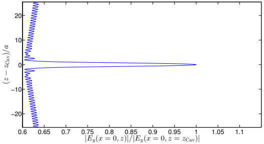


$Q_3 = 1.6 \cdot 10^3$



$Q_4 = 2.0 \cdot 10^4$

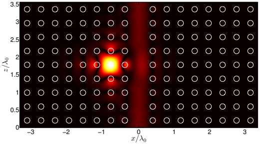


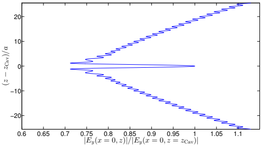


Quasi-Normal Modes in Side-Coupled PhC Cavities

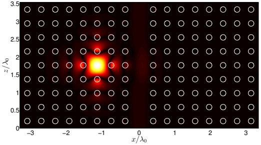
$$Q = \omega_{\mathrm{R}}/\gamma$$

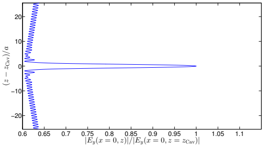
$$Q_2 = 1.5 \cdot 10^2$$





$$Q_3 = 1.6 \cdot 10^3$$





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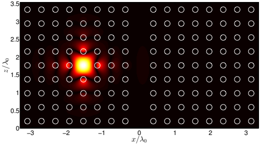
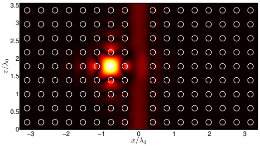
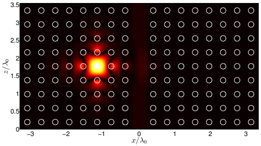


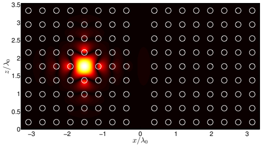
Quasi-Normal Modes in Side-Coupled PhC Cavities

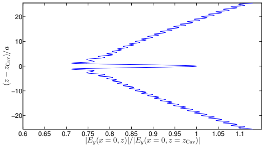
$$Q = \omega_{\mathrm{R}}/\gamma$$

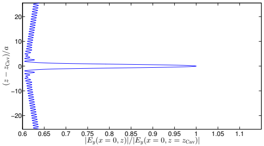
$Q_2 = 1.5 \cdot 10^2$ | $Q_3 = 1.6 \cdot 10^3$ | $Q_4 = 2.0 \cdot 10^4$

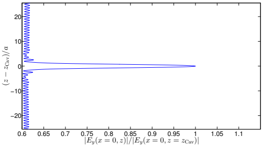






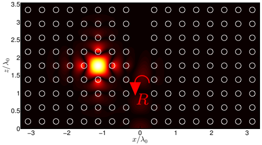






Conclusions

Open nanophotonic resonators
support leaky optical modes



Conclusions

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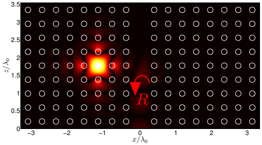
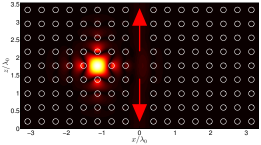
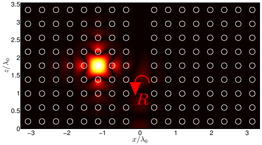


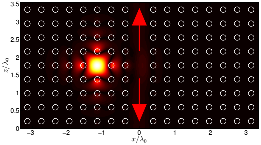
Quasi-normal modes as a rigorous framework for open resonators



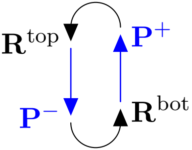
Open nanophotonic resonators support leaky optical modes

Quasi-normal modes as a rigorous framework for open resonators



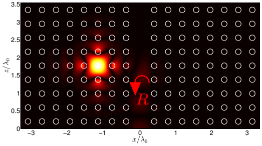


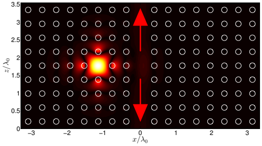
$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z)$$



Open nanophotonic resonators support leaky optical modes

Quasi-normal modes as a rigorous framework for open resonators





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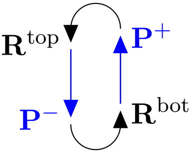
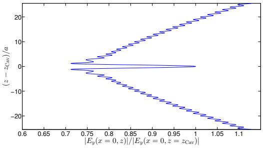


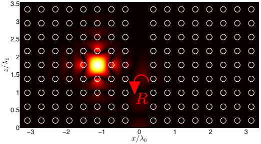
$$Q = \omega_{\mathrm{R}}/\gamma$$

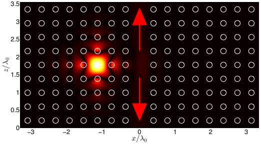


Conclusions

Open nanophotonic resonators support leaky optical modes

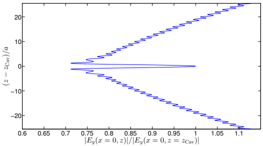
Quasi-normal modes as a rigorous framework for open resonators

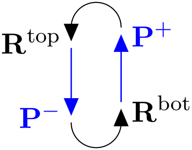




$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z)$$

$$Q = \omega_\mathrm{R}/\gamma$$





J. R. de Lasson

P. T. Kristensen

J. Mørk

N. Gregersen