A Bloch Mode Expansion Approach for Analyzing Quasi-Normal Modes in Open Nanophotonic Structures

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META'14, May 23 2014

DTU Fotonik
Department of Photonics Engineering

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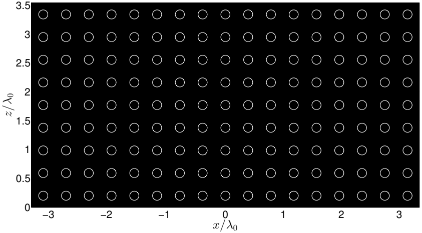
Scattering on Open Photonic Resonator – Modes?



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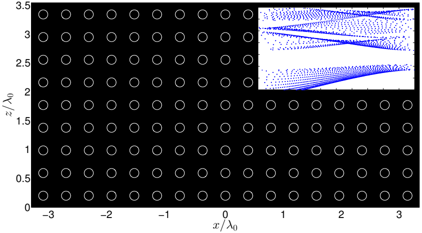

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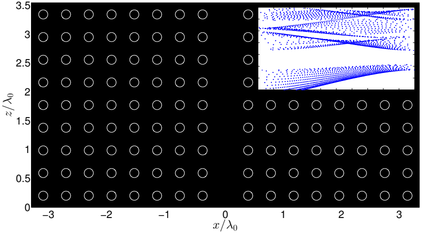

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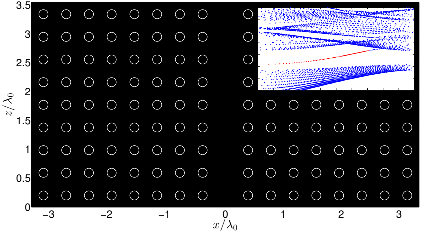

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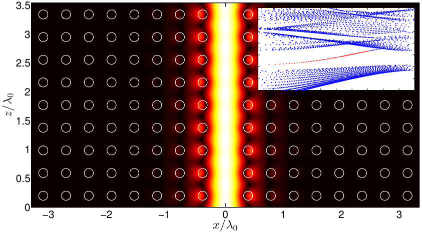

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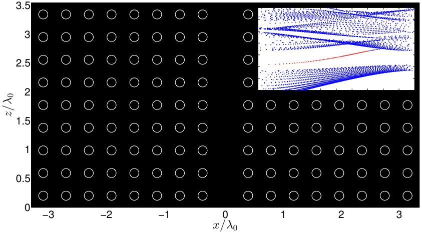

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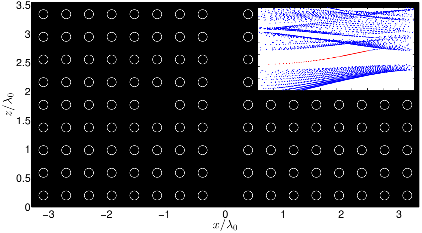

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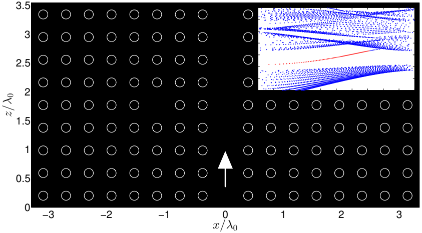

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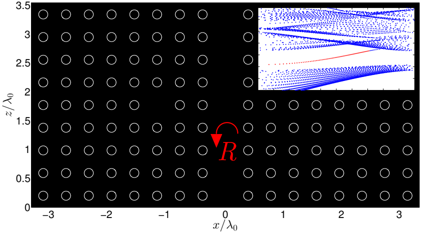
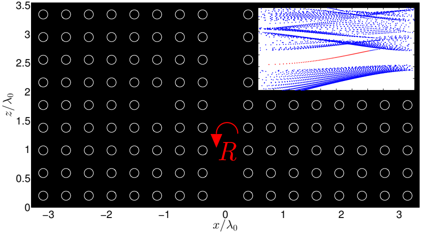
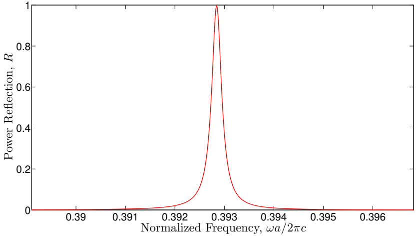
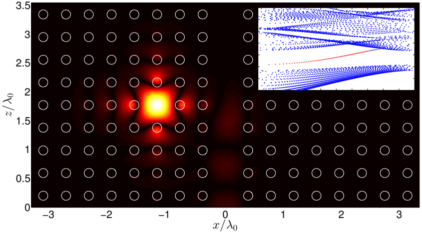
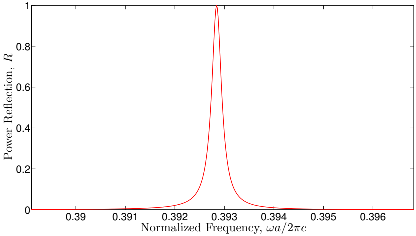



R
z/λ_0
x/λ_0
3.5
3
2.5
2
1.5
1
0.5
0
−3
−2
−1
0
1
2
3


Power Reflection, R
Normalized Frequency, $\omega a/2\pi c$
1
0.8
0.6
0.4
0.2
0
0.39
0.391
0.392
0.393
0.394
0.395
0.396

Scattering on Open Photonic Resonator – Modes?





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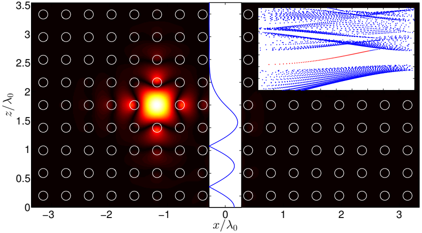

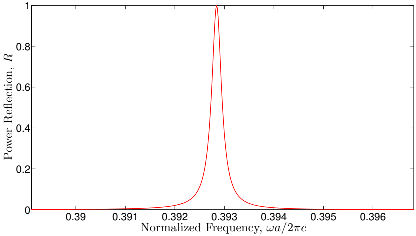

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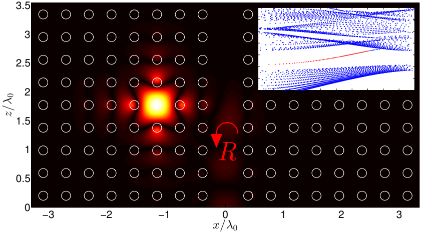

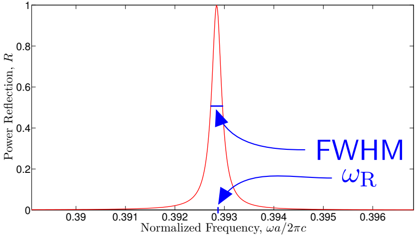

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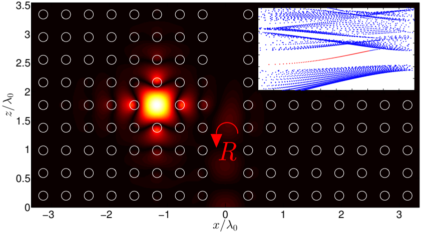

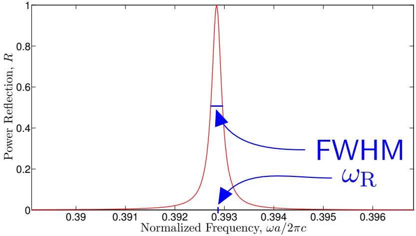

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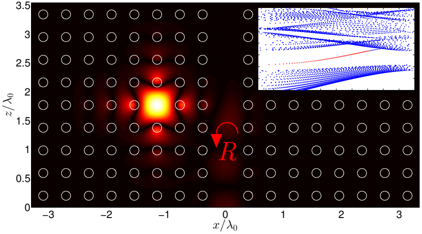
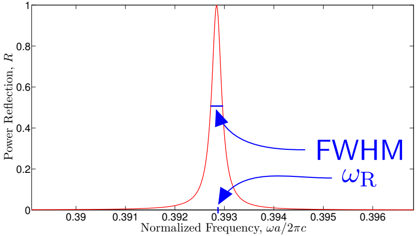
Implicit assumption of the existence of a leaky resonator mode





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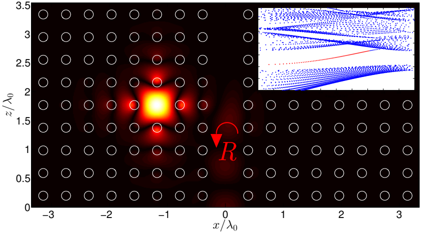
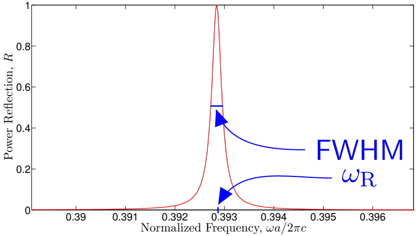
$$\mathbf{E}(\mathbf{r};t) = \mathbf{E}(\mathbf{r})\exp\left(-\mathrm{i}\tilde{\omega}t\right), \qquad \tilde{\omega} \equiv \omega_\mathrm{R} - \mathrm{i}\gamma$$





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...How to formalize the description of these modes?

Outline

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- Definition of and previous work on quasi-normal modes

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- Roundtrip matrix technique for calculating quasi-normal modes

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- Quasi-normal modes in photonic crystal cavities side-coupled to W1 waveguide

Quasi-Normal Modes: Definition

Definition: Time-harmonic solutions $\mathbf{E}(\mathbf{r};t) = \mathbf{E}(\mathbf{r};\omega)\exp(-\mathrm{i}\omega t)$ of $\nabla\times\nabla\times\mathbf{E} = \left(\frac{\omega}{\mathrm{c}}\right)^2\epsilon\mathbf{E}$

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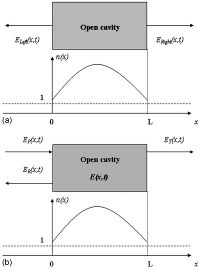
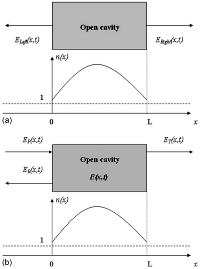
Explicit description of time-decaying resonator mode.



[A. Settimi *et al.*, J. Opt. Soc. Am. B **26**, 876-891 (2009)]

Quasi-Normal Modes: Selection of Previous Work



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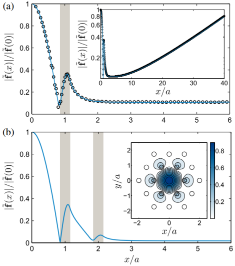

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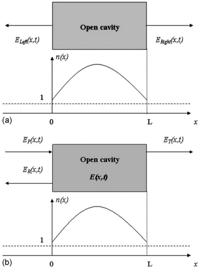

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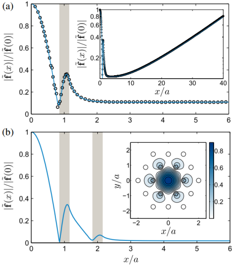

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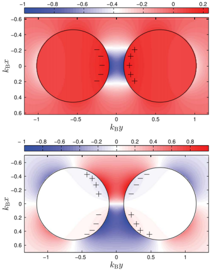

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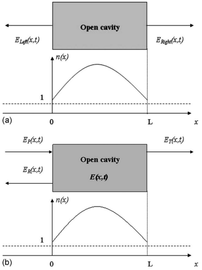

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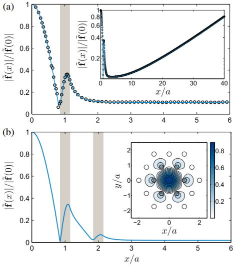

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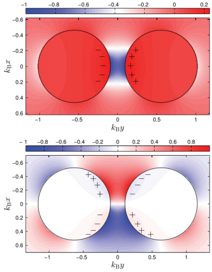

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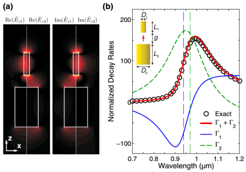

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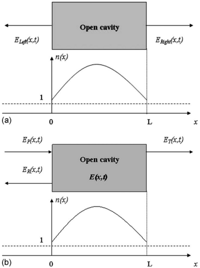

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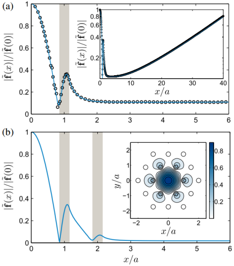

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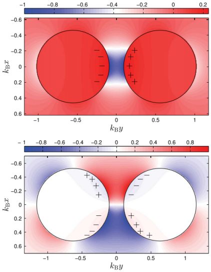

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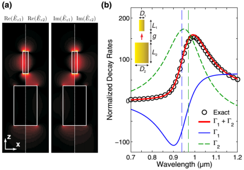

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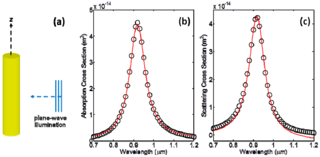
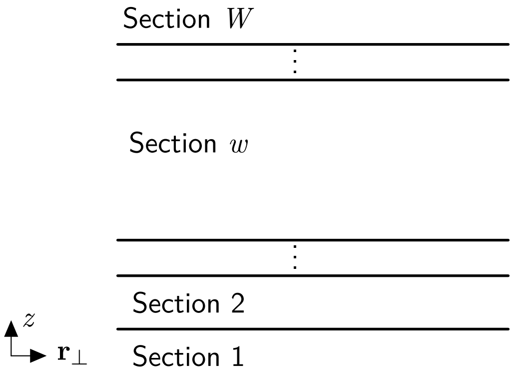


[Q. Bai *et al.*, Opt. Express **21**, 27371-27382 (2013)]

Bloch Mode Expansions: Calculating Quasi-Normal Modes



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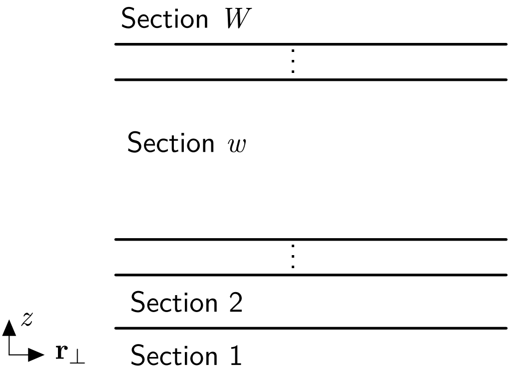
$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z),$$



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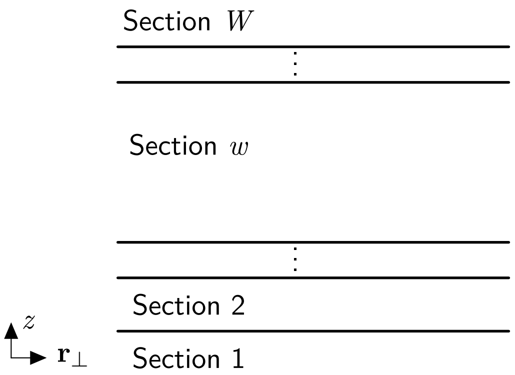

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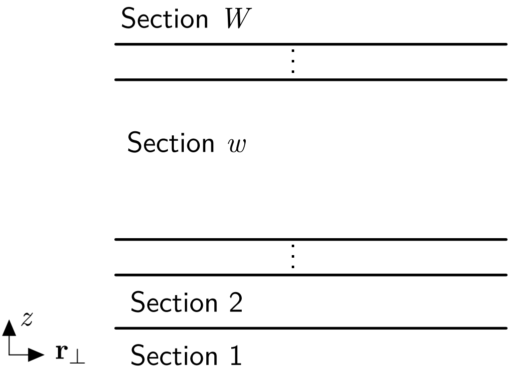

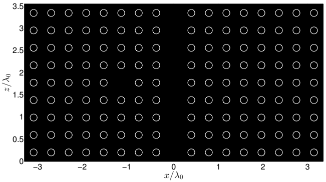

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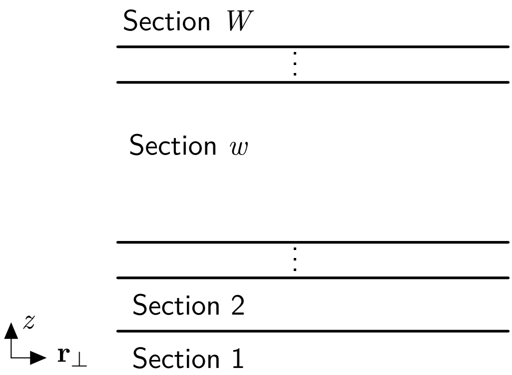

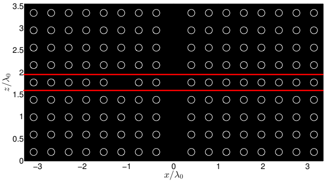

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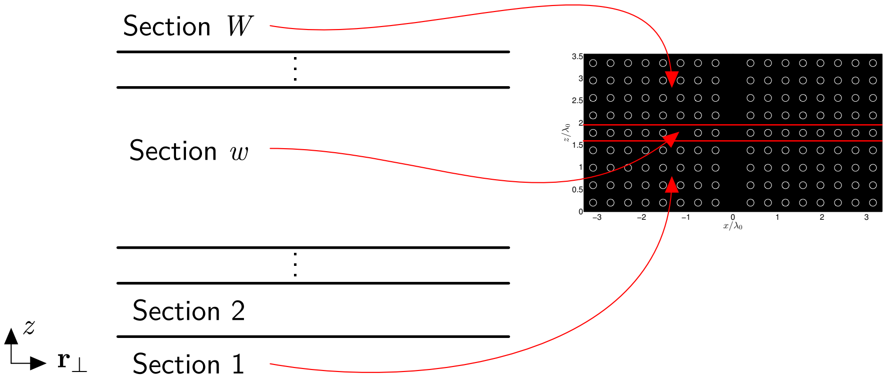

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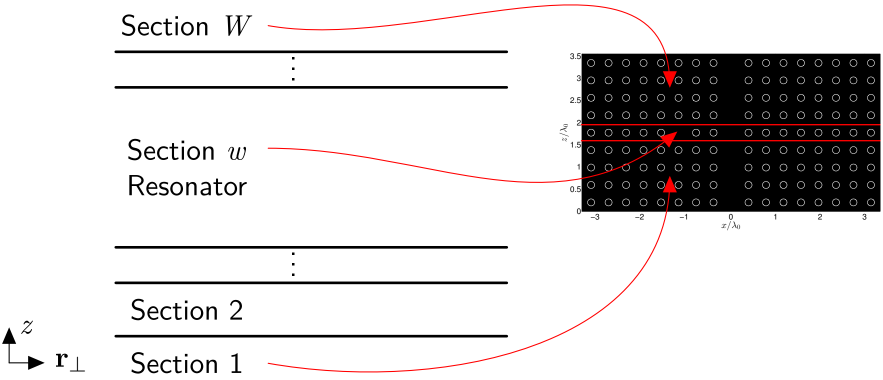

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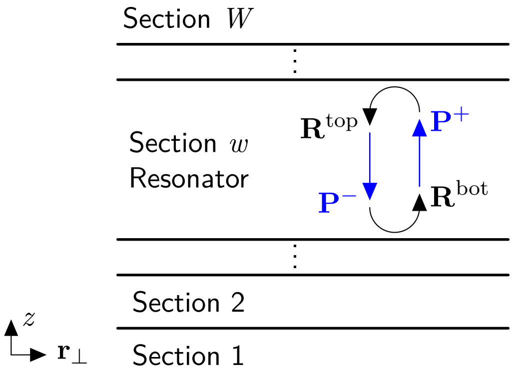

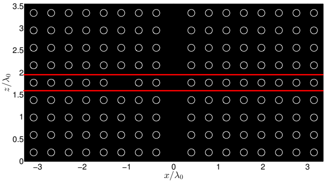

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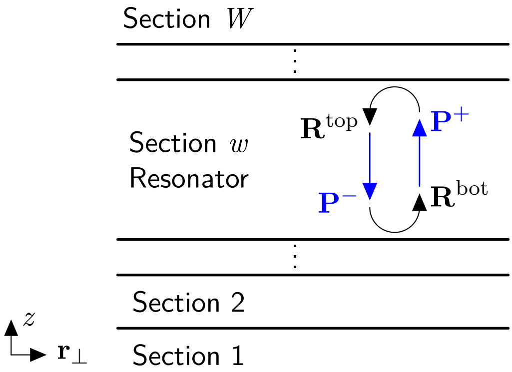

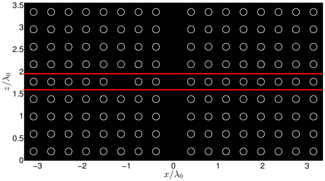

Quasi-normal mode condition:

Resonator roundtrip matrix $\mathbf{M}(\omega) \equiv \mathbf{R}^{\mathrm{bot}}\mathbf{P}^{-}\mathbf{R}^{\mathrm{top}}\mathbf{P}^{+}$ satisfying

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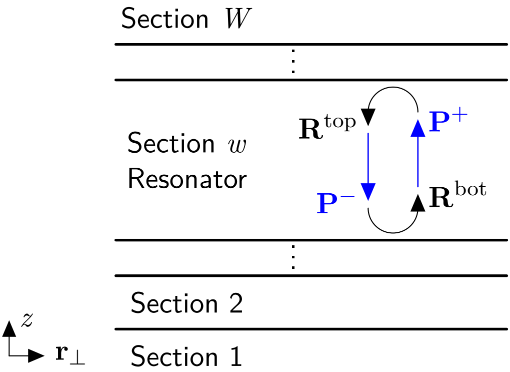
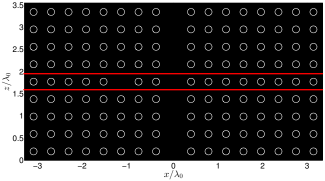




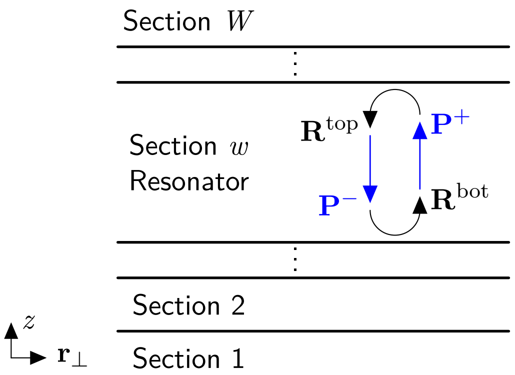
Quasi-normal mode condition:

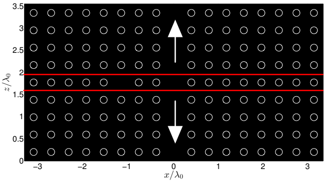
Resonator roundtrip matrix $\mathbf{M}(\omega) \equiv \mathbf{R}^{\mathrm{bot}}\mathbf{P}^{-}\mathbf{R}^{\mathrm{top}}\mathbf{P}^{+}$ satisfying

$$\mathbf{M}(\omega)\mathbf{c}^w = \lambda_{\mathrm{R}}\mathbf{c}^w, \quad \lambda_{\mathrm{R}} = 1$$

[J. R. de Lasson *et al.*, arXiv:1405.2595]

$$\mathbf{E}^w(\mathbf{r}) = \sum_j c_j^w \mathbf{e}_j^w(\mathbf{r}_\perp, z), \qquad \mathbf{e}_j^w(\mathbf{r}_\perp, z+a) = \exp\left(\mathrm{i}k_j^w a\right) \mathbf{e}_j^w(\mathbf{r}_\perp, z)$$





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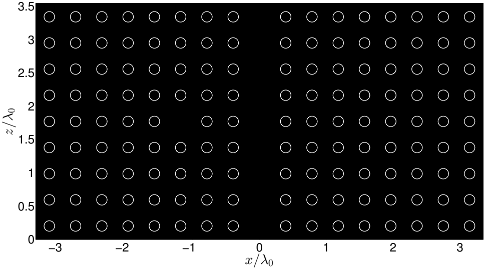
Quasi-Normal Modes in Side-Coupled PhC Cavities



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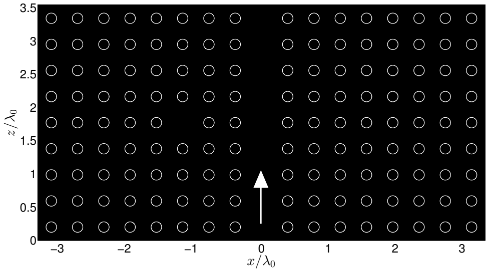

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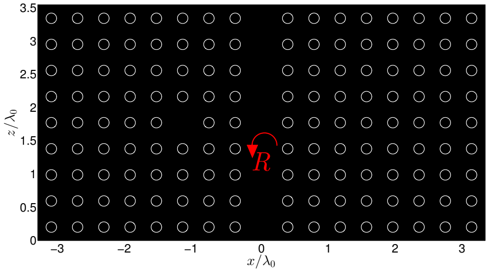

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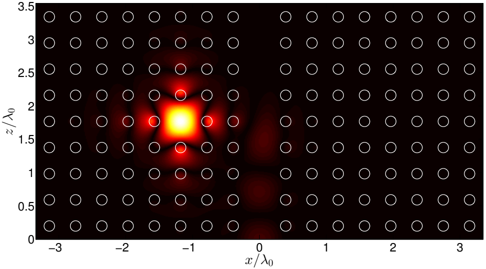

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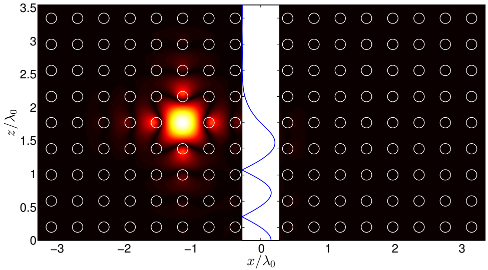

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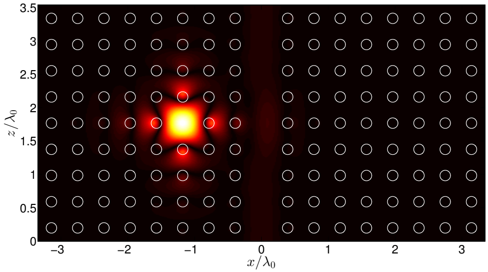

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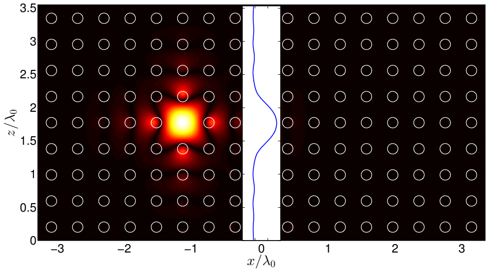

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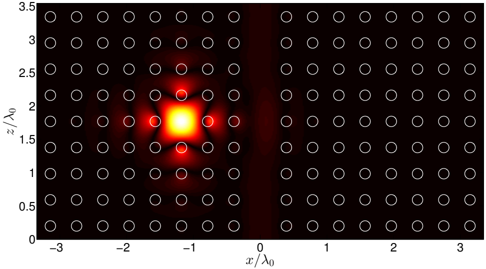

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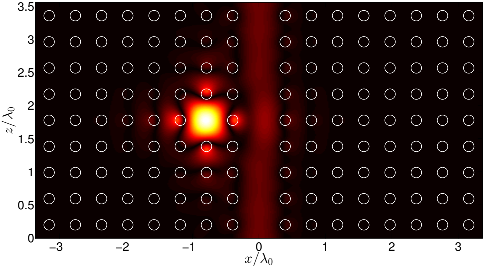

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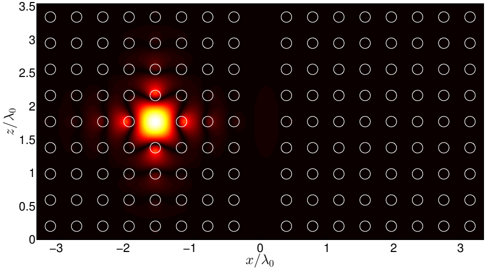

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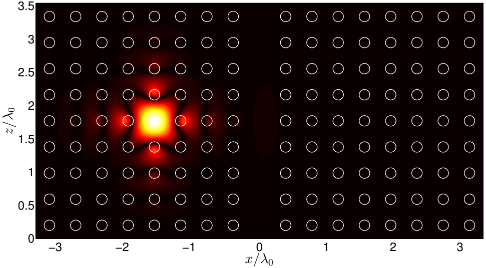
Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \frac{\omega_R}{2\gamma}$$

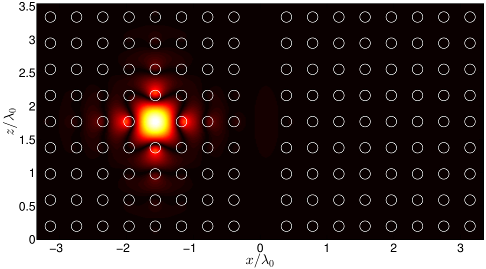


[J. R. de Lasson *et al.*, arXiv:1405.2595]

Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \frac{\omega_\mathrm{R}}{2\gamma}$$

$$Q_{4a} = 2.0 \cdot 10^4$$



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Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \frac{\omega_{\mathrm{R}}}{2\gamma}$$

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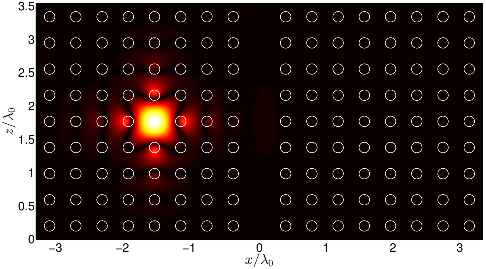

[J. R. de Lasson *et al.*, arXiv:1405.2595]

Quasi-Normal Modes in Side-Coupled PhC Cavities

$$Q = \frac{\omega_R}{2\gamma}$$

$$Q_{2a} = 1.5 \cdot 10^2$$

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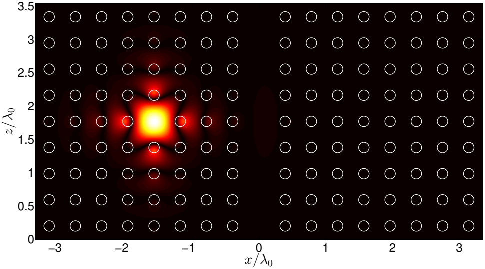

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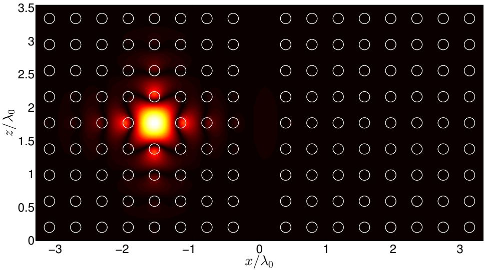

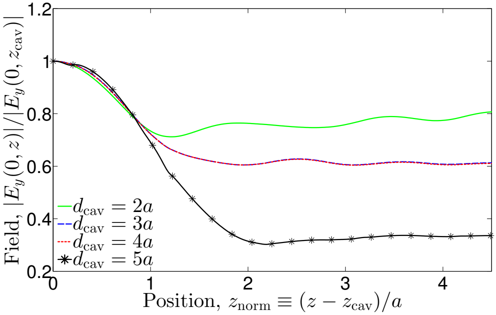

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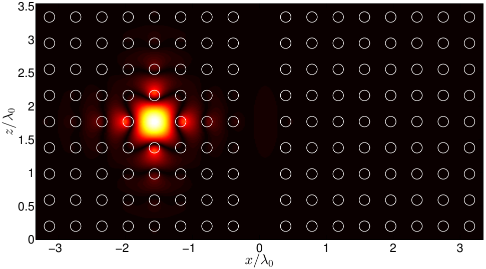

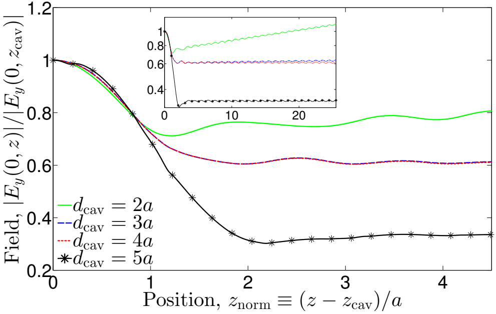

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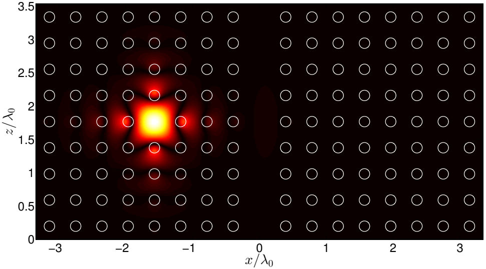

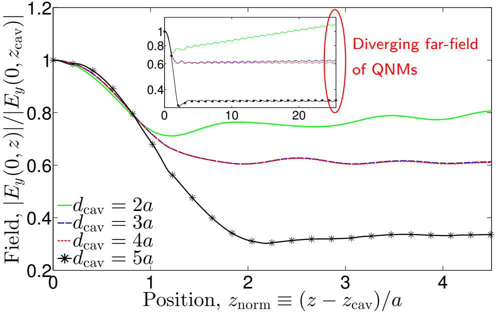

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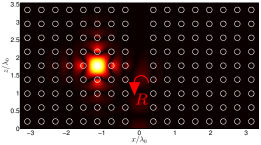
Conclusions

Open nanophotonic resonators
support leaky optical modes



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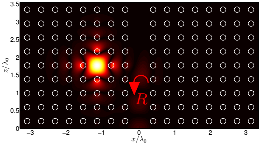
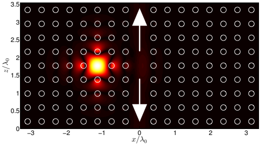


Quasi-normal modes as a rigorous framework for open resonators



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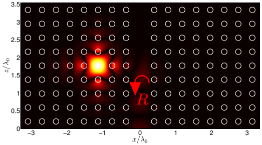

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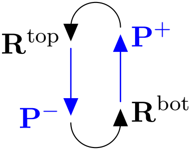
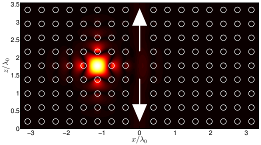


Quasi-normal modes as a rigorous framework for open resonators



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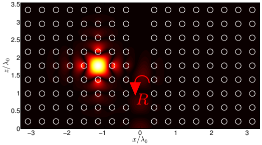

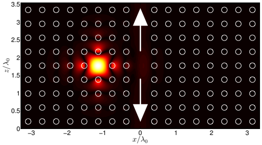

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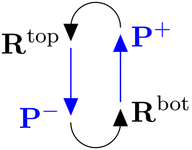

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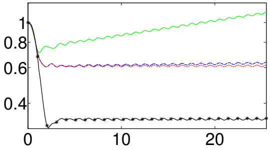

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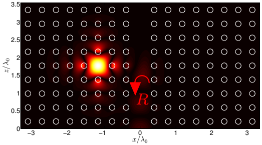

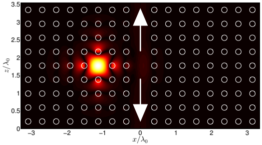

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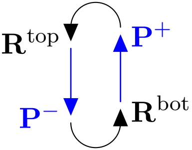

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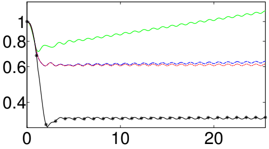

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