Optical Simulations in an Open Geometry Using the Eigenmode Expansion Technique

Thomas Christensen & Jakob Rosenkrantz de Lasson

DTU Fotonik
Department of Photonics Engineering

5 July 2010
Introduction

Outline of Thesis

- Modeling of the $\mathbf{E}$-field in slab structures illuminated by TE-waves.
- Two approaches: The closed and the open geometry.
- Field profiles and the normalized spontaneous emission rate used to assess the two approaches.
Wave Equations

Maxwell’s Equations → Scalar Helmholtz Equation

- Starting point: Maxwell’s equations. 3D-problem.
- Reduction to a 1D-problem:
  - No free charges/currents and constant material parameters give Helmholtz equation:
    \[ \nabla^2 E + n^2 k_0^2 E = 0. \]
  - Slab structures: \( E(x, y, z) \equiv E(x, z) \).
  - TE-waves polarized along \( y \):
    \[ \nabla^2 E_y(x, z) + n^2 k_0^2 E_y(x, z) = 0. \]
  - Separation of variables, \( E_y(x, z) = e_x(x) e_z(z) \), yields solutions:
    \[ e_x(x) \propto \exp(\pm i\kappa x), \quad e_z(z) \propto \exp(\pm i\beta z). \]
The Closed Geometry Approach

Boundary Conditions

- Main characteristic: Finite $x$-width. Metallic boundaries.
- Boundary conditions (BCs):
  - Inner BCs:
    - Continuity and differentiability of field in all points.
  - Outer BCs:
    - Vanishing field at $x = 0$ and $x = L_x$: $e_x(0) = e_x(L_x) = 0$.
    - $z$-BCs: Illumination.

- The $x$-BCs are handled by the **semi-analytical approach** and ensure a discrete, complete and orthonormal set of eigenmodes.
- The $z$-BCs are handled by the **scattering matrix formalism**.
The Closed Geometry Approach

Eigenmodes

- Two types of eigenmodes:
  - Guided modes: 
    \[ n_2^2 k_0^2 > \beta_k^2 > n_1^2 k_0^2. \]
  - Semi-radiating modes: 
    \[ n_1^2 k_0^2 > \beta_l^2 > -\infty. \]

- Eigenmode expansion: 
  \[ E_{\text{closed}} \sim \sum_j e_{x,j} e_{z,j}. \]
The Closed Geometry Approach

Field Profile: Abruptly Terminated Waveguide

Abruptly terminated waveguide illuminated by fundamental mode.

Small solution domain.
Parasitic reflections!
The Closed Geometry Approach
Field Profile: Abruptly Terminated Waveguide

Abruptly terminated waveguide illuminated by fundamental mode.

Large solution domain.
The Closed Geometry Approach

Field Profile: Bragg Grating

Bragg grating illuminated by fundamental mode.

Grating dimensions tuned to reflection at $\lambda = 1.55 \, \mu m$. 
The Closed Geometry Approach

Spontaneous Emission Rate: Theory

- The normalized spontaneous emission rate (SER):
  \[ \alpha \equiv \frac{\gamma}{\gamma_0}. \]

- Equivalent quantity:
  \[ \alpha = \frac{P}{P_0}. \]

- The power emitted from a dipole is proportional to the field due to the dipole, evaluated in the position of the dipole:
  \[ P = -\frac{1}{2} \text{Re}(E(x_c, z_c)). \]

- The normalized SER should, in principle, be calculable by increasing \( L_x \) until convergence.
The Closed Geometry Approach

Spontaneous Emission Rate: Results

Single-layers with uniformity along $z$.

Vacuum layer

Waveguide layer

No convergence!
Three-layer structure. Waveguide in vacuum.

Extension to multi-layered structures straightforward.
The Open Geometry Approach

Presentation

- No outer $x$-BCs: $E(x, z)$ is nowhere forced to vanish.
- More distinct mode types:
  - Guided modes: $U_j(x)$. Discrete set, finite number.
  - Radiation modes: $\phi_l(x, s)$ or $\psi_m(x, \rho)$. Continuum.
- The eigenmode expansion:

\[
E_{\text{closed}} \sim \sum_j e_{x,j} \rightarrow E_{\text{open}} \sim \sum_j U_j + \sum_{m=1}^{2} \int_0^{\infty} \psi_m(\rho) \, d\rho.
\]

Different treatment of radiation modes.
The Open Geometry Approach
Spontaneous Emission Rate in Single-Layer Structure: Theory

- As in closed geometry: $P = -\frac{1}{2} \text{Re}(E(x_c, z_c))$. Field due to dipole required.
- The power emitted in a uniform layer has an analytical solution:
  \[
  P_0 = \frac{1}{8}.
  \]
- $P_0$ is the reference power in all SER-computations.
- Power in waveguide single-layer structures require approximation of improper integrals, cf. eigenmode expansion.
The Open Geometry Approach

Spontaneous Emission Rate in Single-Layer Structure: Results

\[ \theta \)-sampling of integrals: Rapidly converging results.\]

Convergence: Relative deviation of 1% with 24 (vacuum) and 7 (waveguide) sampling points.

\[ \alpha \]
The Open Geometry Approach
Two Layers: Abruptly Terminated Waveguide

Reflection and transmission between waveguide and uniform layer.
The Open Geometry Approach

Reflection and Transmission Coefficients

The fields in each layer, $E_1(x, z)$ and $E_2(x, z)$, given by the eigenmode expansion:

$$E_1(x, z) = U_1(x) \left[ \exp(-i\beta_1 z) + R_1 \exp(i\beta_1 z) \right]$$

$$+ \sum_{m=1}^{2} \int_0^\infty R(\rho) \psi_m(x, \rho) \exp(i\beta(\rho)z) \, d\rho,$$

$$E_2(x, z) = \sum_{l=1}^{2} \int_0^\infty T(s) \phi_l(x, s) \exp(-i\gamma(s)z) \, ds.$$

Reflection and transmission coefficients, $R_1$, $R(\rho)$, and $T(s)$, to be determined.
The Open Geometry Approach

Aperture Field

- Continuity and differentiability of the field at $z = 0$ and use of orthonormality and completeness relations gives a Fredholm Equation of the Second Kind for the aperture field, $\Phi(x)$:

$$\Phi(x) = \Phi_0(x) + \lambda \int_{-\infty}^{\infty} \Phi(x') K(x, x') \, dx'.$$

- $R_1, R(\rho), \text{ and } T(s)$: Functions of $\Phi(x)$.

- Assuming uniform convergence, an approximate solution to the integral equation is a truncated Liouville-Neumann series:

$$\Phi_N(x) = \Phi_0(x) + \lambda^N \sum_{j=1}^{N} C_j(x).$$
The Open Geometry Approach
First Order Reflection and Transmission Coefficients

Evanescent regime: Exponentially decaying along the propagation direction. Near-field.

Propagating regime: Far-field.
The Open Geometry Approach
First Order Field Profile

- No parasitic reflections.
- Radiation modes in the waveguide layer: Near-field effect.
Conclusion

- The closed geometry approach:
  - Illustrative field profiles, but parasitic reflections.
  - Fluctuating SER results: Relative deviation of 8%-26%. Primarily due to semi-radiating modes.
  - Versatile tool for modeling of arbitrary structures.

- The open geometry approach:
  - Rapidly converging single-layer SER-results using $\theta$-sampling. Relative deviation of 1% with 24 and 7 samples.
  - Abruptly terminated waveguide:
    - Incident guided mode: Field profile without parasitic reflections.
    - Incident radiation mode: Further work needed.
  - SER calculations in arbitrary open geometry possible when treatment of incident radiation mode is complete.

Open geometry: More accurate results.