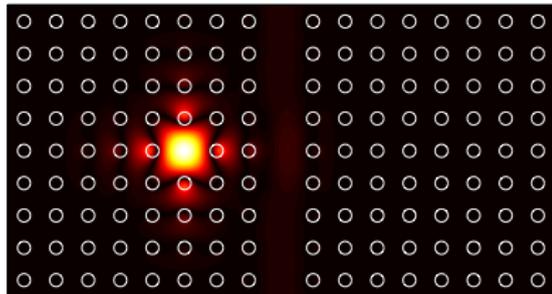


Modeling and Simulations of Light Emission and Propagation in Open Nanophotonic Systems

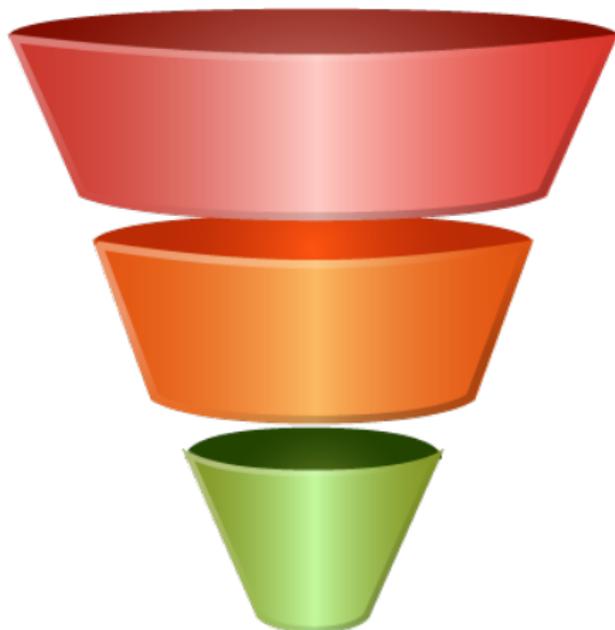


Jakob Rosenkrantz de Lassen

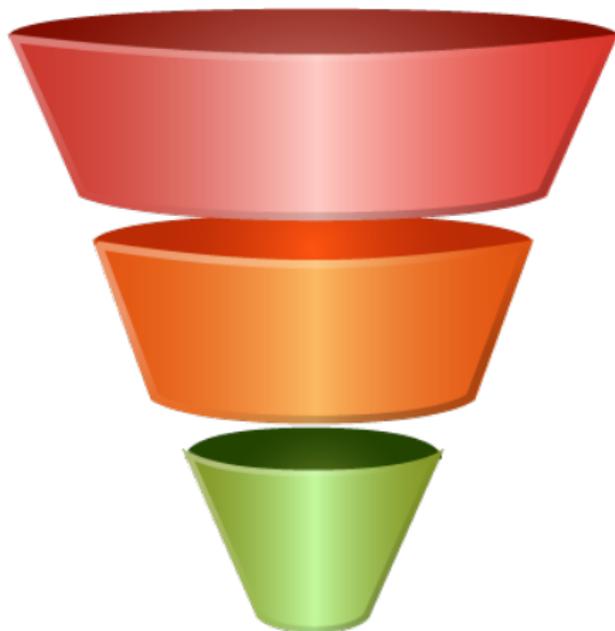
Nanophotonics Theory
& Signal Processing

www.nanophotonics.dk

Ph.D. Lecture, December 16 2015

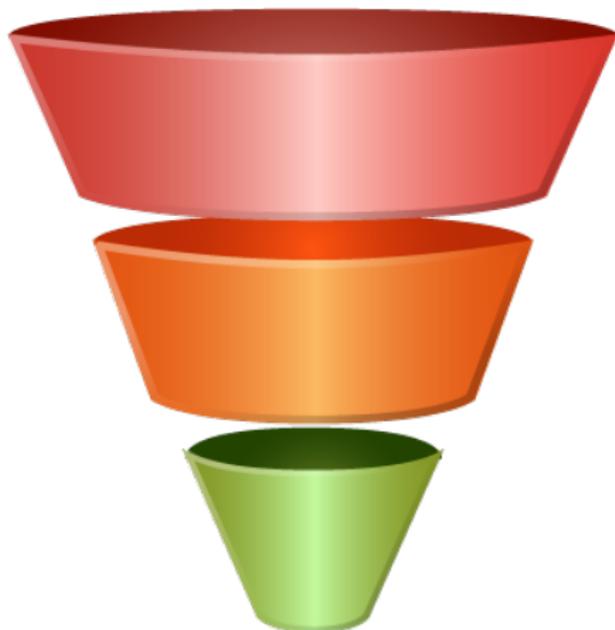


I. *Why*



I. *Why*

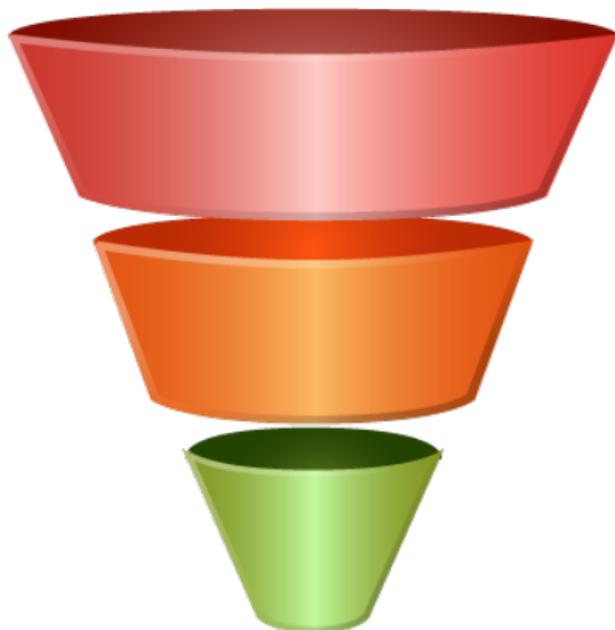
II. *How*



I. *Why*

II. *How*

III. *What*

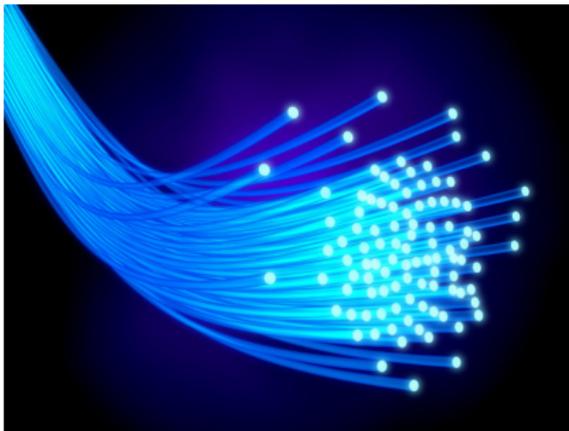


I. *Why* do we do our research?



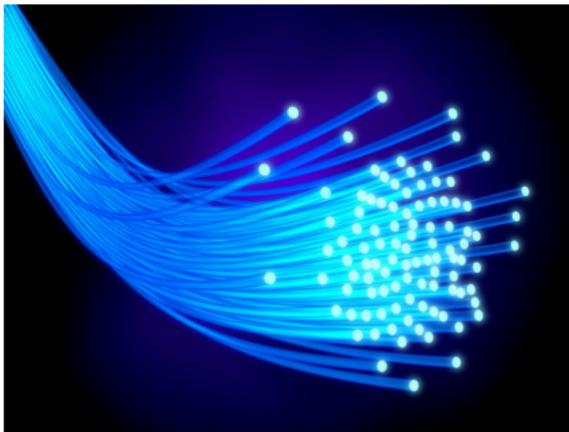






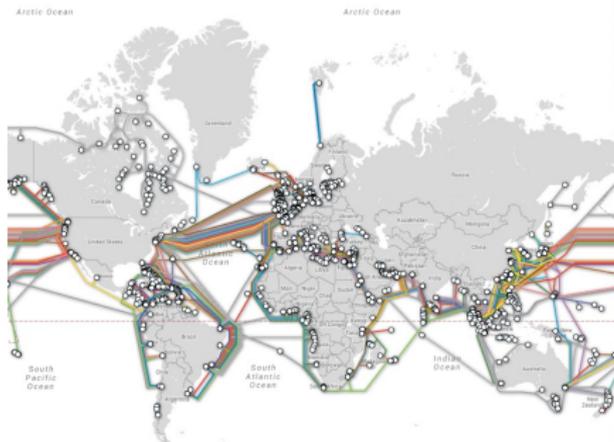
Vidensklumme: Verdens største maskine kører på lys

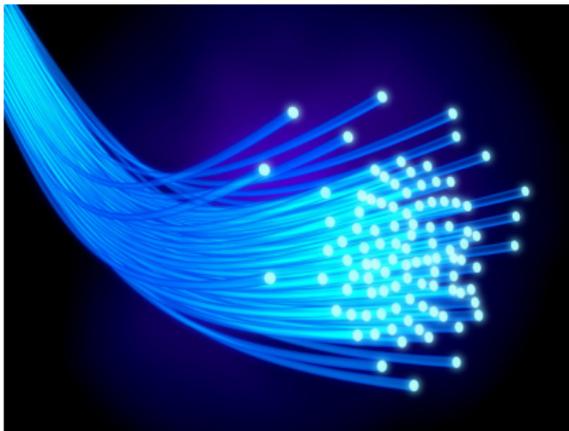
Internettet vokser med 35-50 procent om året, og havde det ikke været for lyset, ville det være brudt sammen for længe siden.



Vidensklumme: Verdens største maskine kører på lys

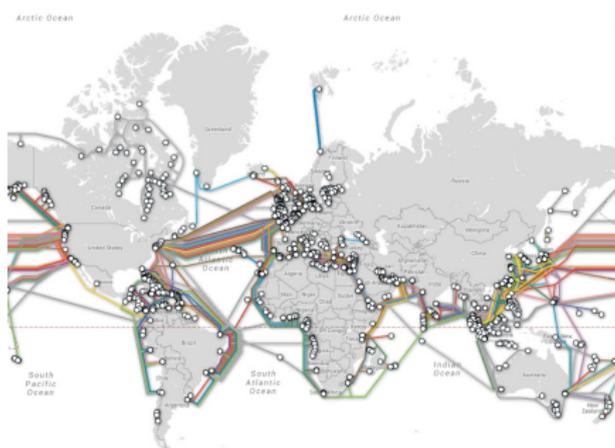
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Vidensklumme: Verdens største maskine kører på lys

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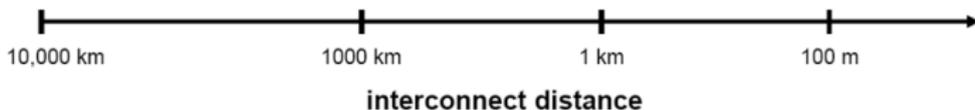
Telecommunications



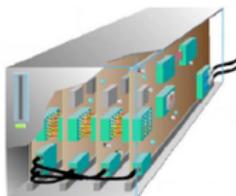
Campus networks



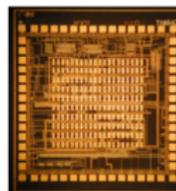
LANs



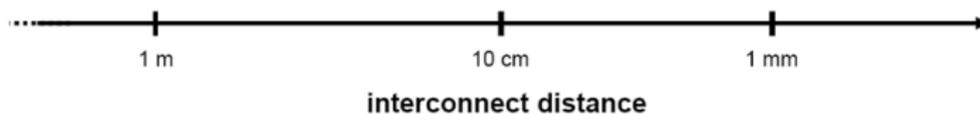
Backplanes & board-to-board



Chip-to-chip



On-chip



[D. A. B. Miller, ICTON 2009]



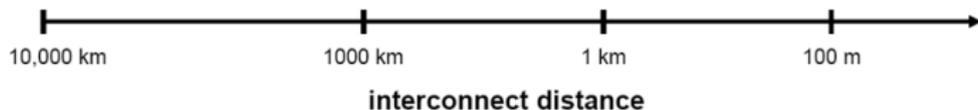
Telecommunications



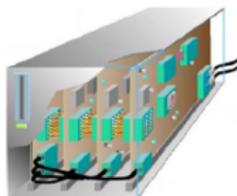
Campus networks



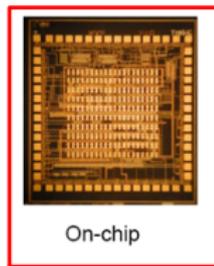
LANs



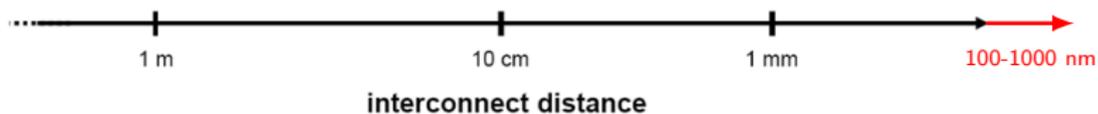
Backplanes & board-to-board



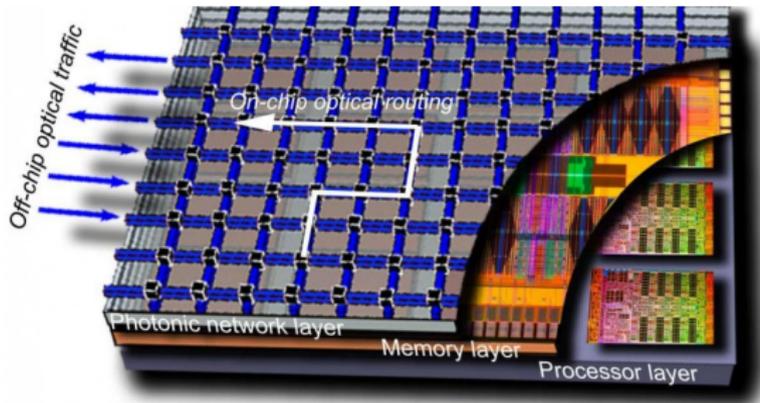
Chip-to-chip



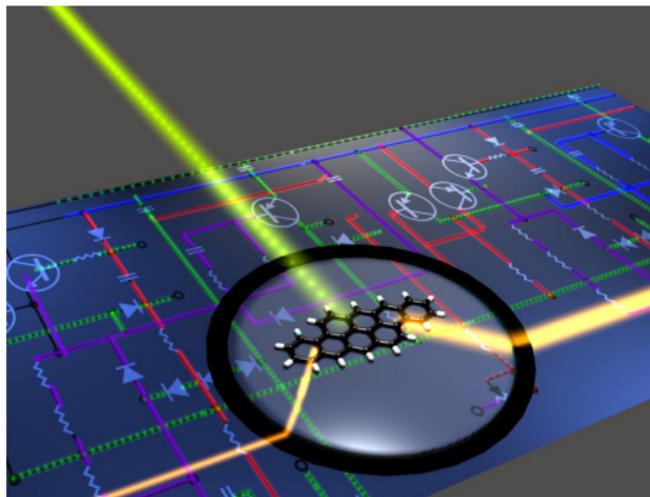
On-chip



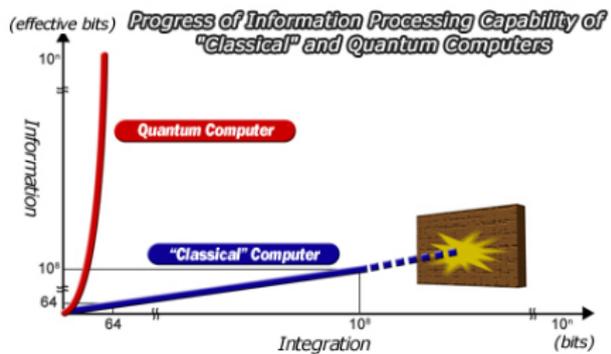
[D. A. B. Miller, ICTON 2009]

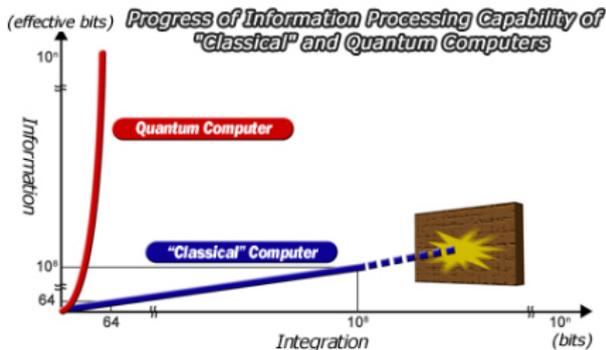


[IBM]



[ETH Zürich]



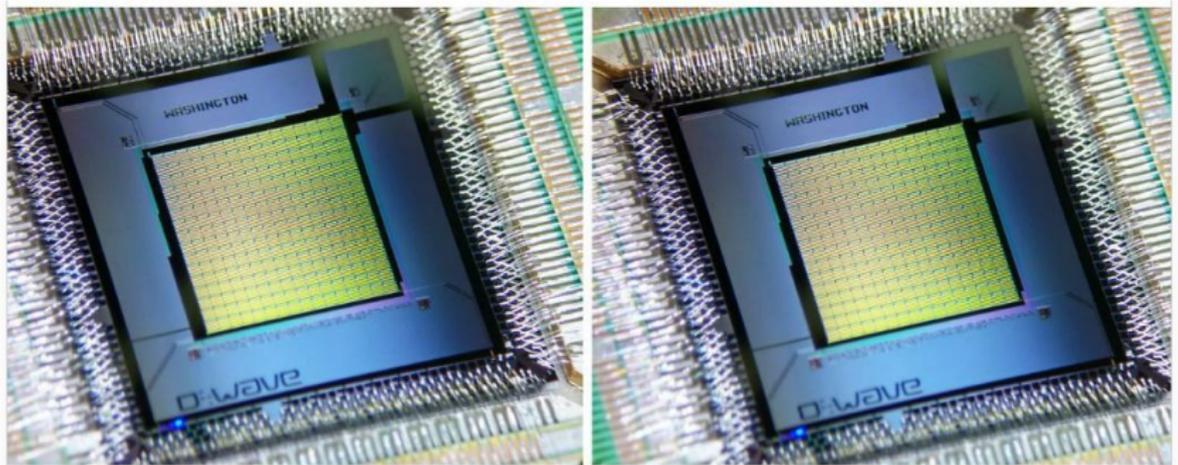


The quantum internet

H. J. Kimble¹

Quantum networks provide opportunities and challenges across a range of intellectual and technical frontiers, including quantum computation, communication and metrology. The realization of quantum networks composed of many nodes and channels requires new scientific capabilities for generating and characterizing quantum coherence and entanglement. Fundamental to this endeavour are quantum interconnects, which convert quantum states from one physical system to those of another in a reversible manner. Such quantum connectivity in networks can be achieved by the optical interactions of single photons and atoms, allowing the distribution of entanglement across the network and the teleportation of quantum states between nodes.

Why IBM and Intel Are Chasing the \$100B Opportunity in Nanophotonics



D-Wave

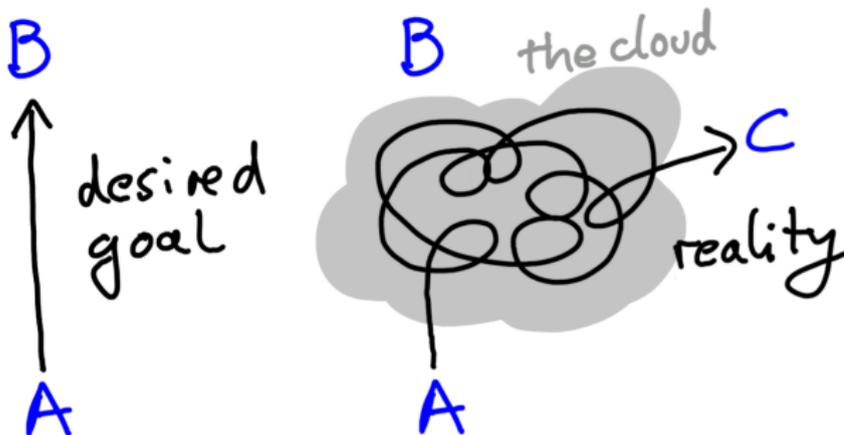
Google's quantum computer is 100 million times faster than your laptop

But is it a true quantum computer?

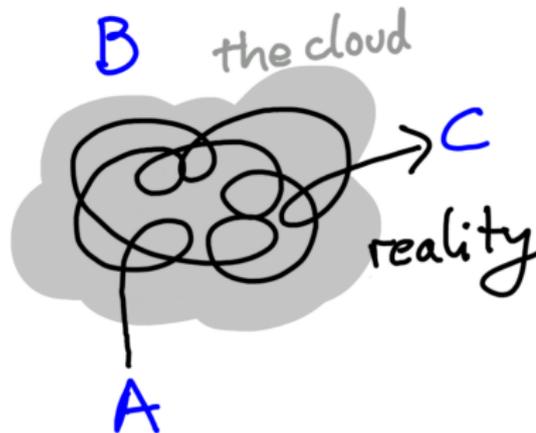
DAVID NIELD 10 DEC 2015



[C. Wilke, "The Serial Mentor"]

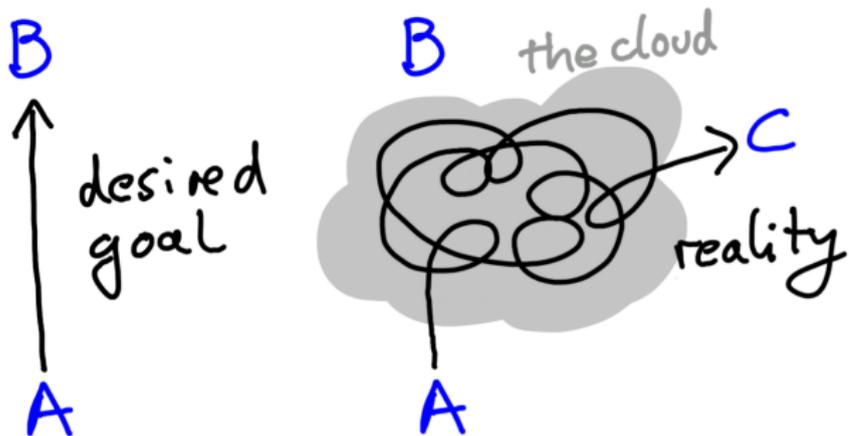


[C. Wilke, "The Serial Mentor"]



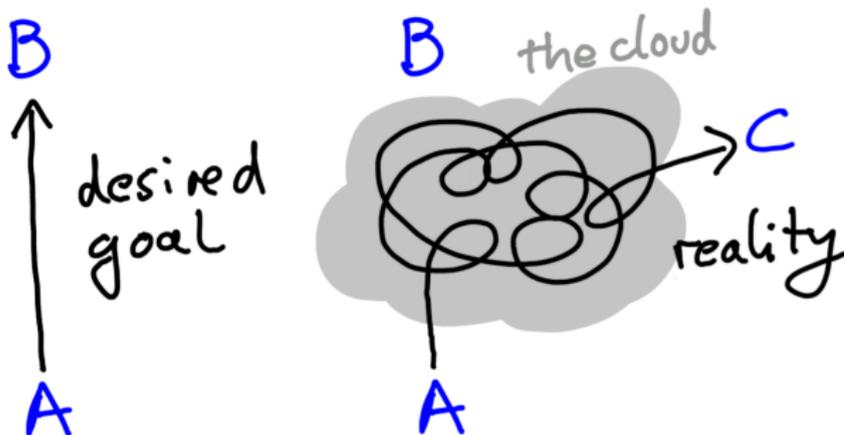
[C. Wilke, "The Serial Mentor"]



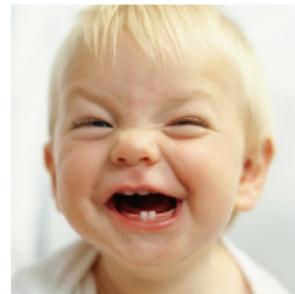


[C. Wilke, "The Serial Mentor"]





[C. Wilke, "The Serial Mentor"]



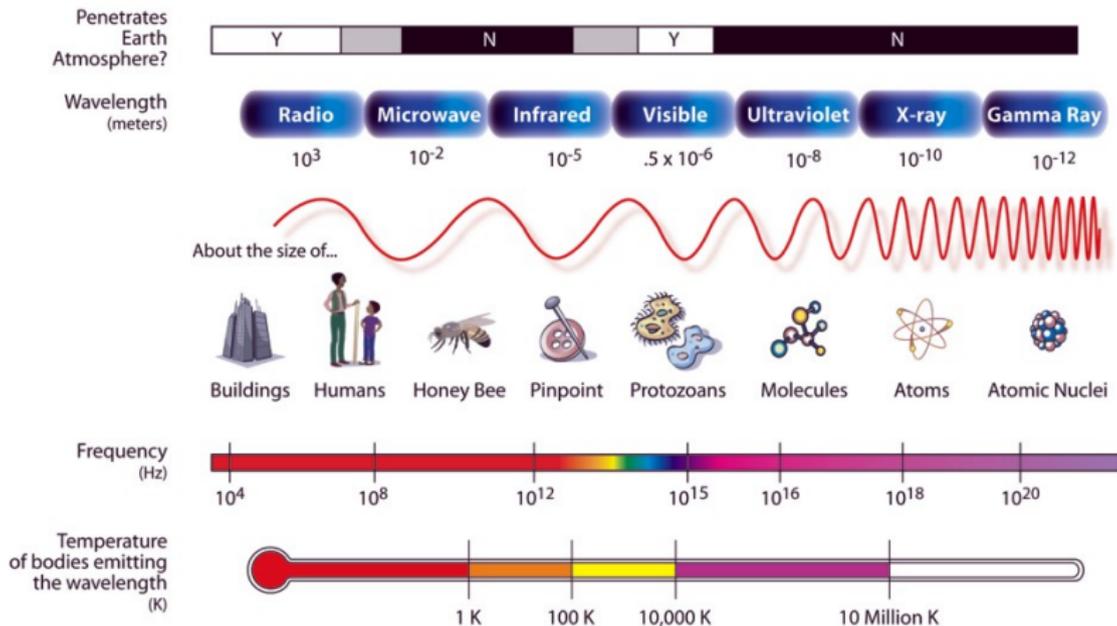
II. *How* do we do our research?

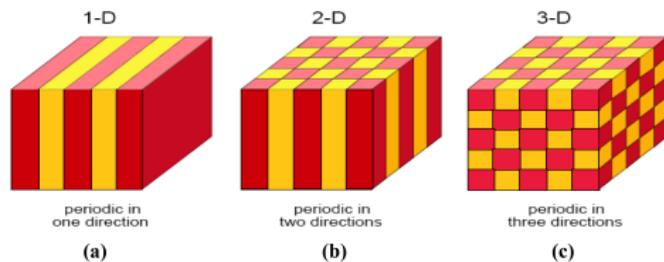
II. *How* do we do our research?

- First half: The structures

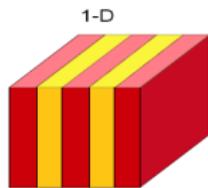


THE ELECTROMAGNETIC SPECTRUM



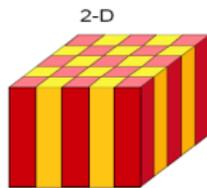


[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]



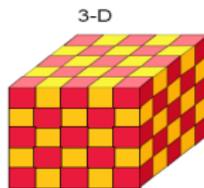
periodic in
one direction

(a)



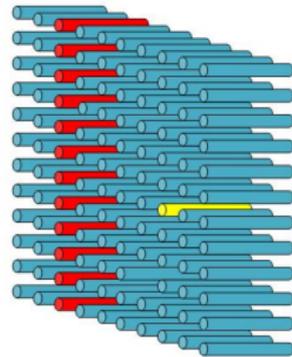
periodic in
two directions

(b)

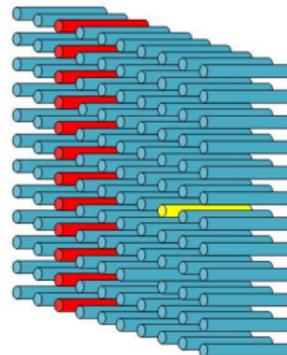
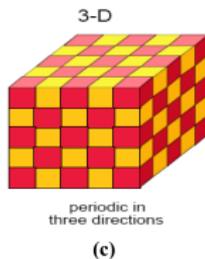
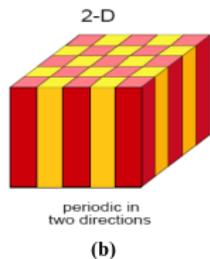
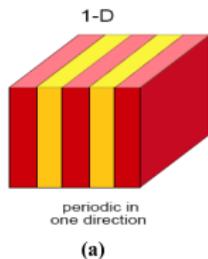


periodic in
three directions

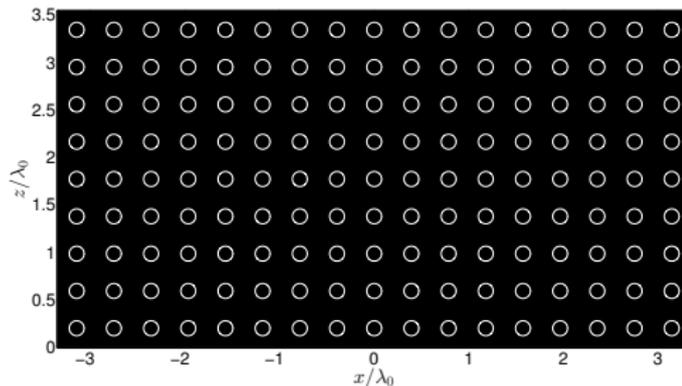
(c)

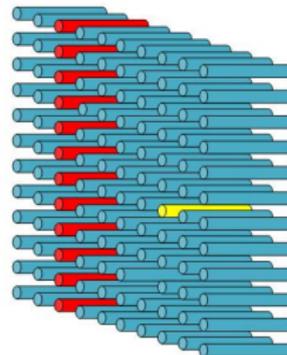
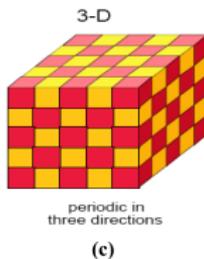
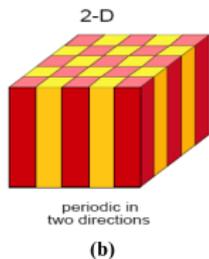
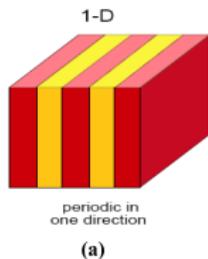


[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]

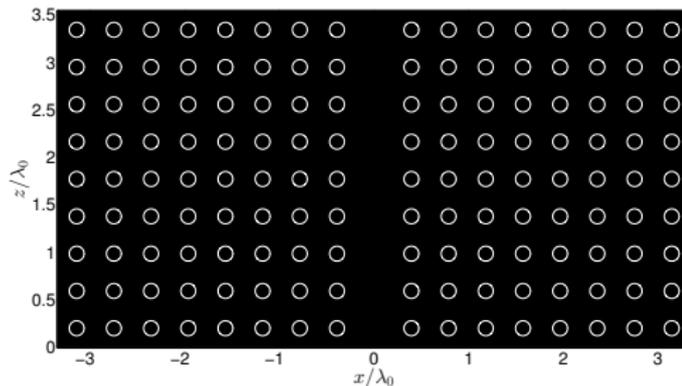


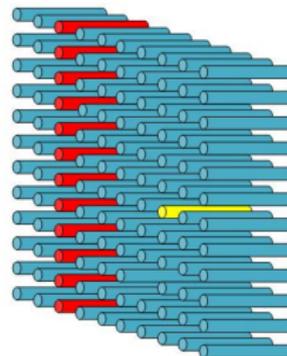
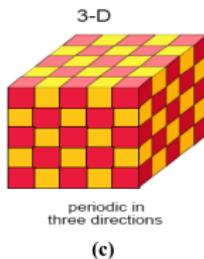
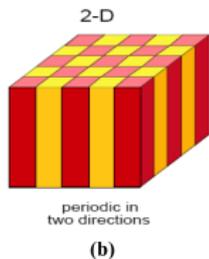
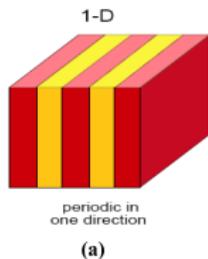
[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]



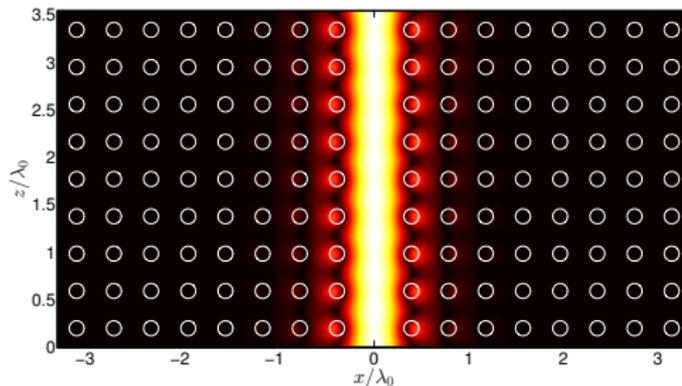


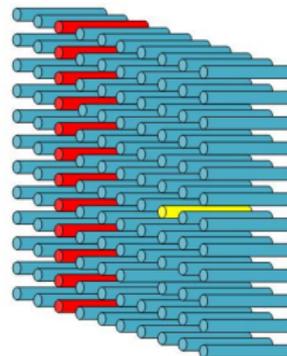
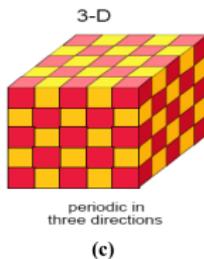
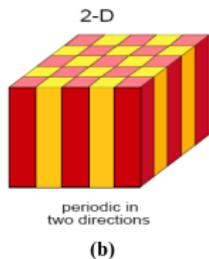
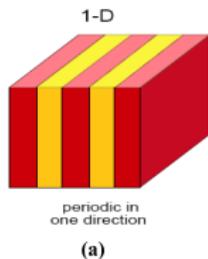
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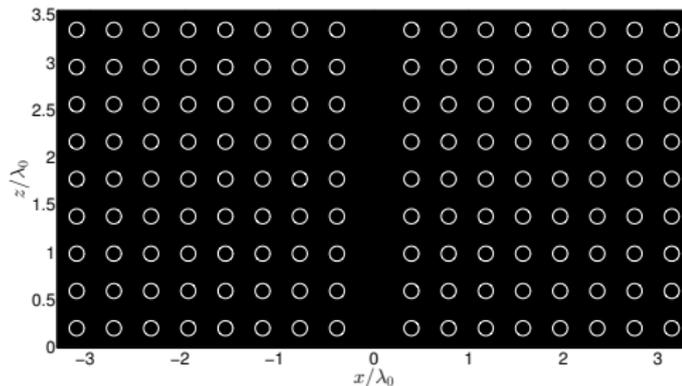


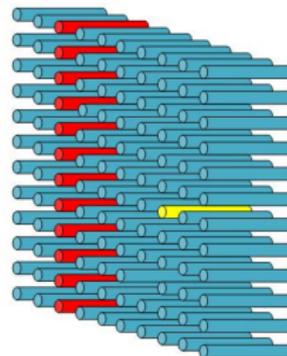
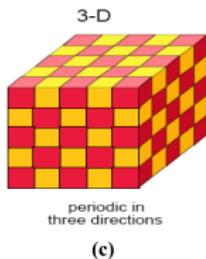
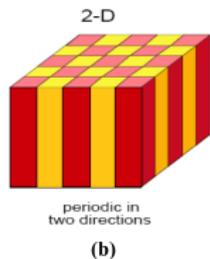
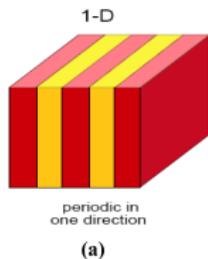
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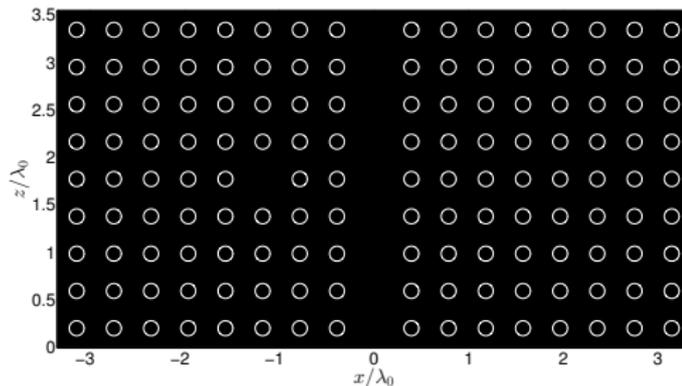


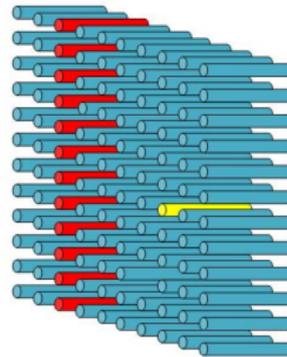
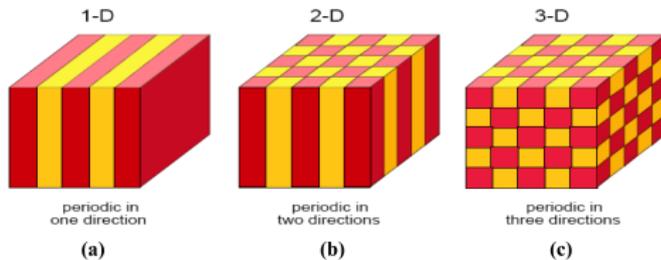
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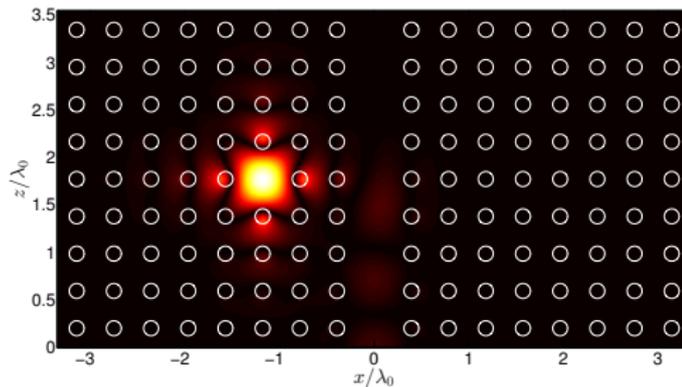


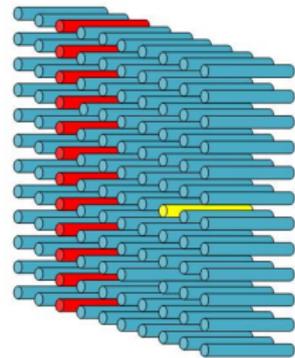
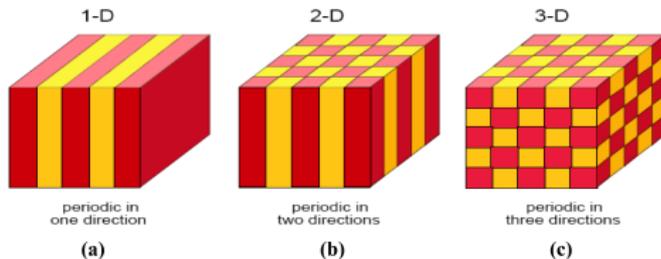
[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]



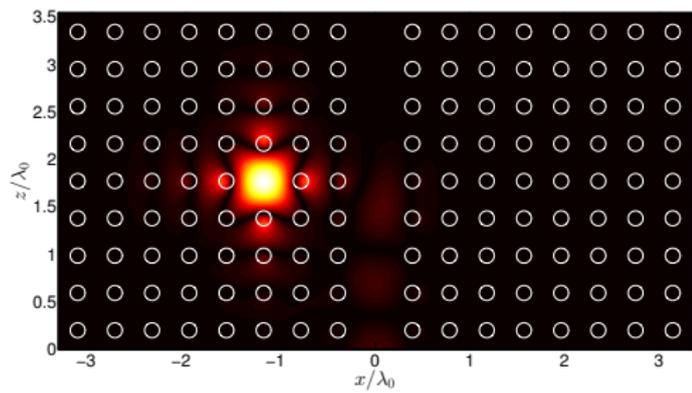


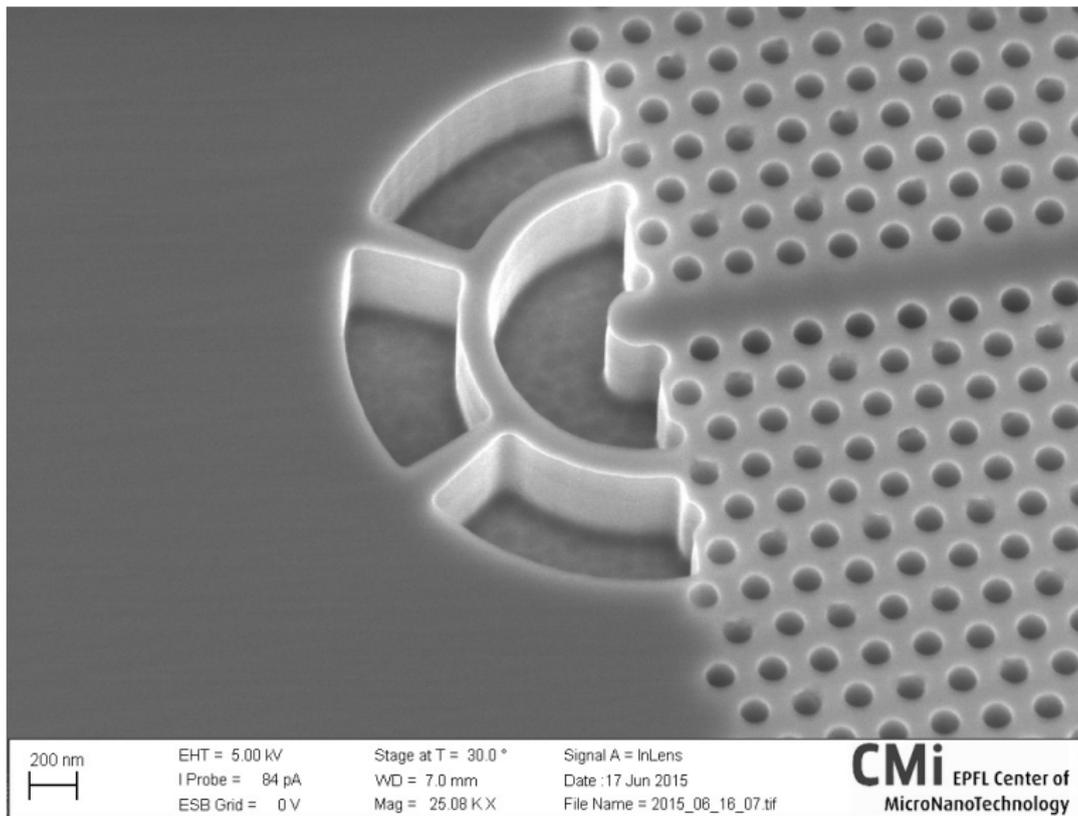
[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]





[J. D. Joannopoulos *et al.*, "Photonic Crystals – Molding the Flow of Light"]





[Courtesy of Laboratory of Physics of Nanostructures, EPFL, Switzerland]

Light speed reduction to 17 metres per second in an ultracold atomic gas

Lene Vestergaard Hau^{*,†}, S. E. Harris[‡], Zachary Dutton^{*,†}
& Cyrus H. Behroozi^{*,§}

[L. V. Hau *et al.*, Nature **397**, 594 (1999)]

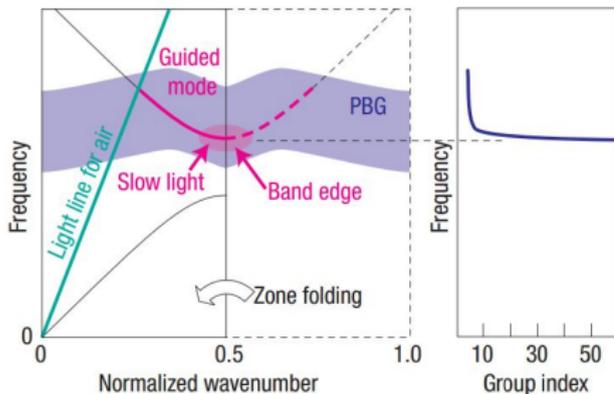


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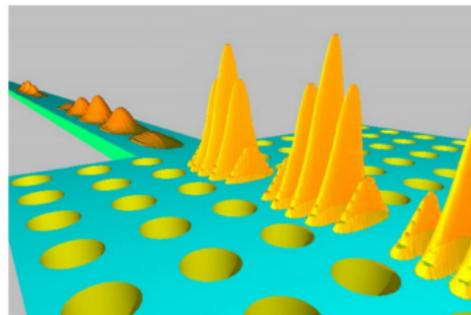


[T. Baba, *Nat. Photonics* **2**, 465 (2008)]

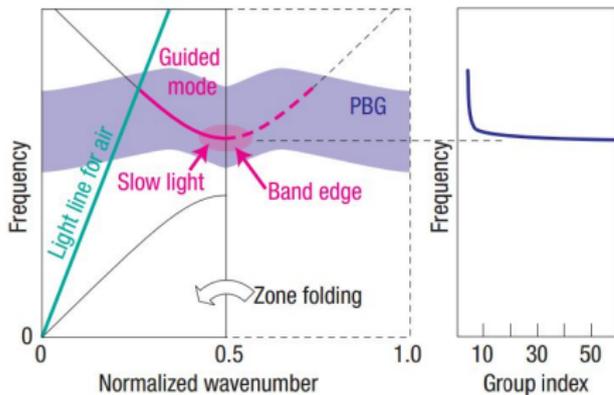
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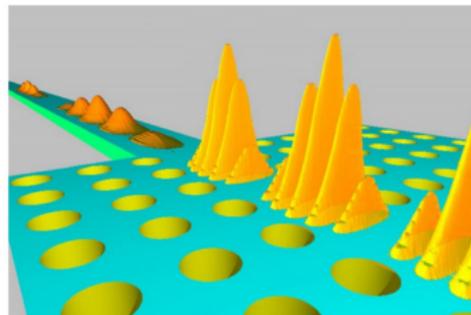


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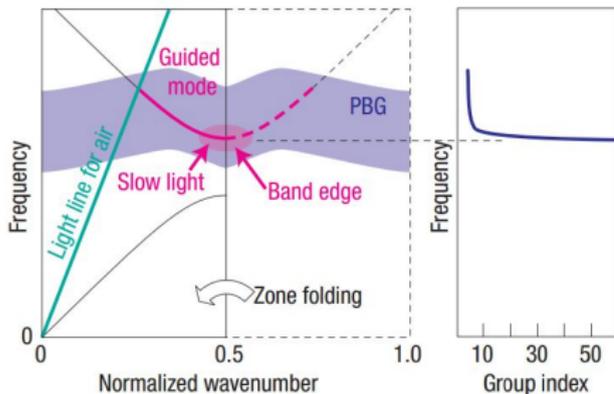
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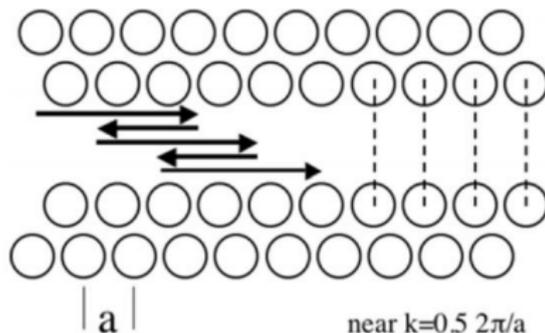
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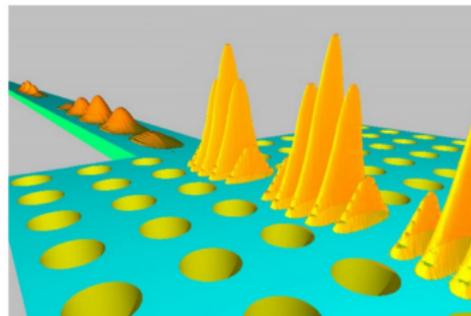
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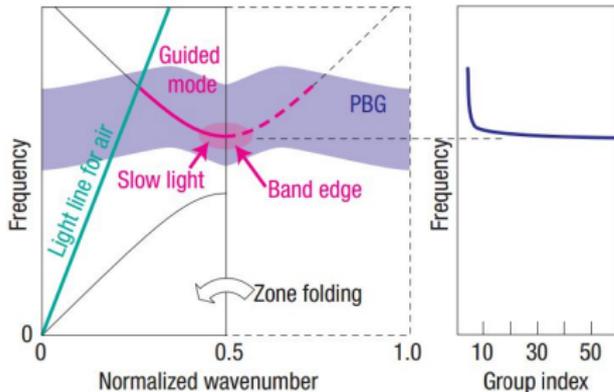
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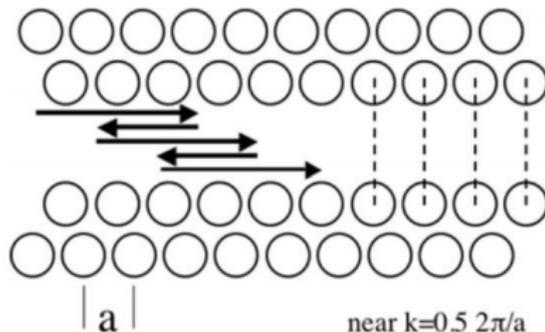
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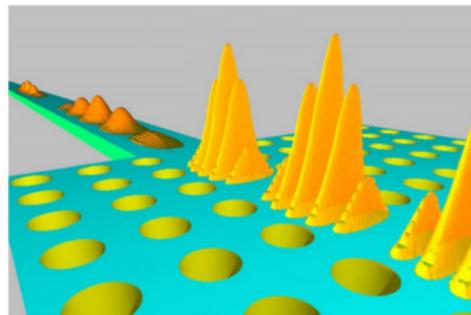
Why do we need slow light?

THOMAS F. KRAUSS

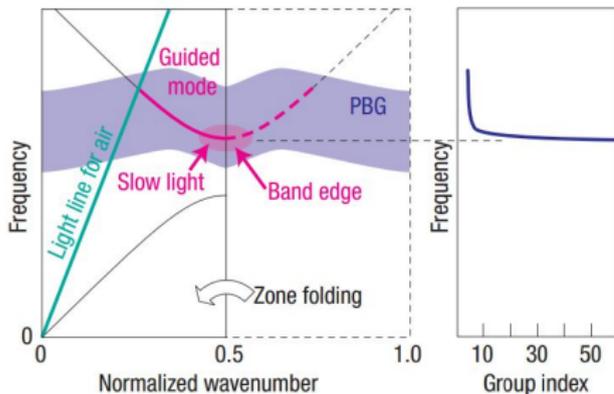
Light speed reduction to 17 metres per second in an ultracold atomic gas

Lene Vestergaard Hau[†], S. E. Harris[‡], Zachary Dutton^{††} & Cyrus H. Behroozi[§]

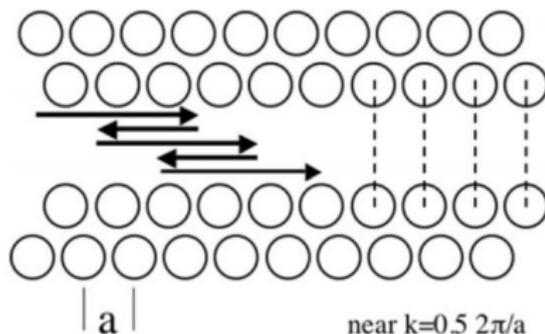
[L. V. Hau *et al.*, *Nature* **397**, 594 (1999)]



[T. F. Krauss, *J. Phys. D* **40**, 2666 (2007)]



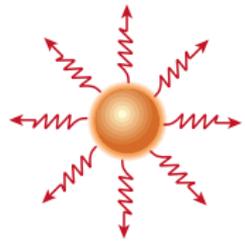
[T. Baba, *Nat. Photonics* **2**, 465 (2008)]



Why do we need slow light?

THOMAS F. KRAUSS

"Because we can" 😊



[K. J. Vahala, Nature **424**, 839 (2003)]

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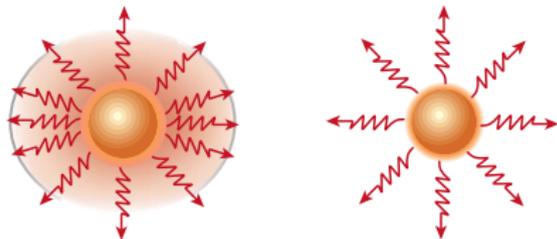
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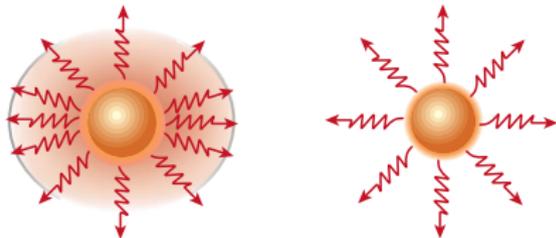
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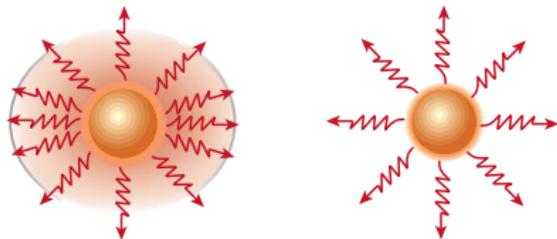
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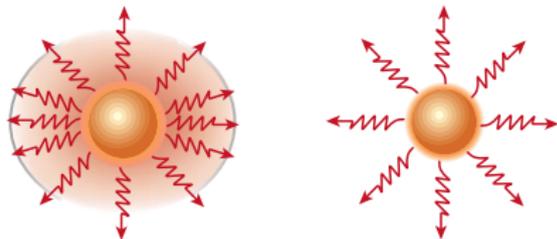
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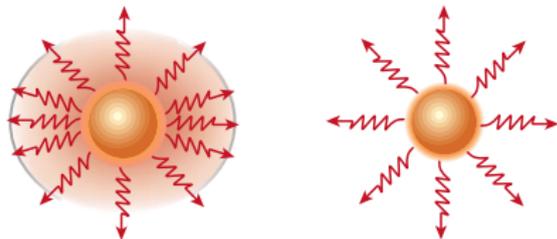
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$$\hat{H}_I = -\hat{\mathbf{p}} \cdot \hat{\mathbf{E}}$$

[L. Novotny and B. Hecht, "Principles of Nano-Optics" (2012)]

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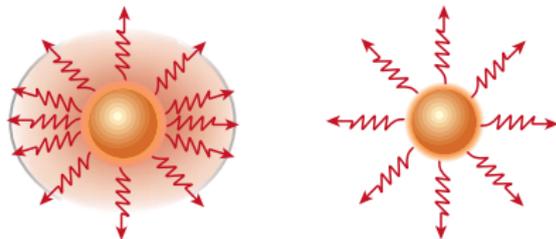
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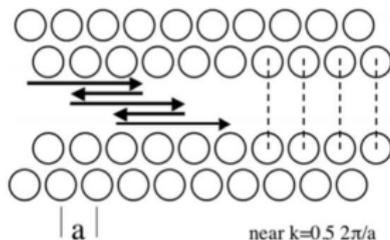
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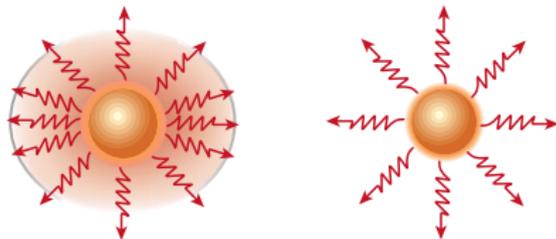
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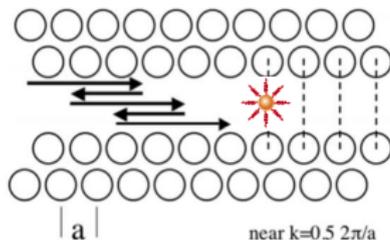
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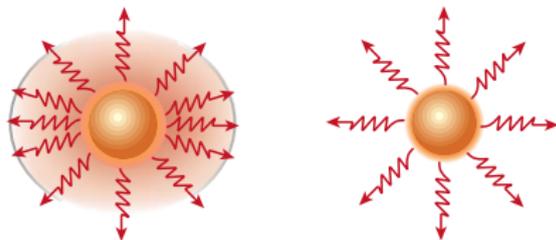
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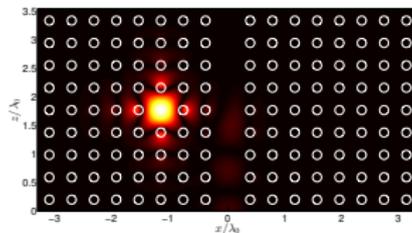
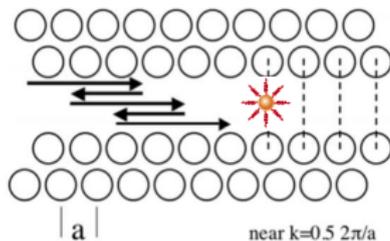
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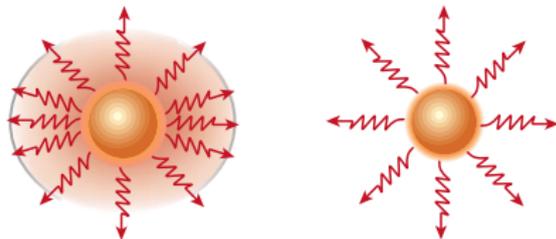
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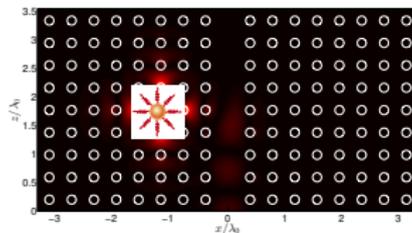
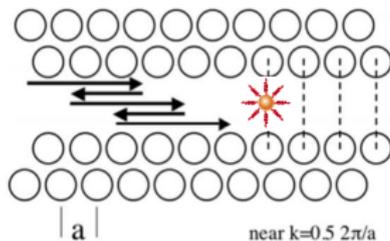
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– First half: The structures

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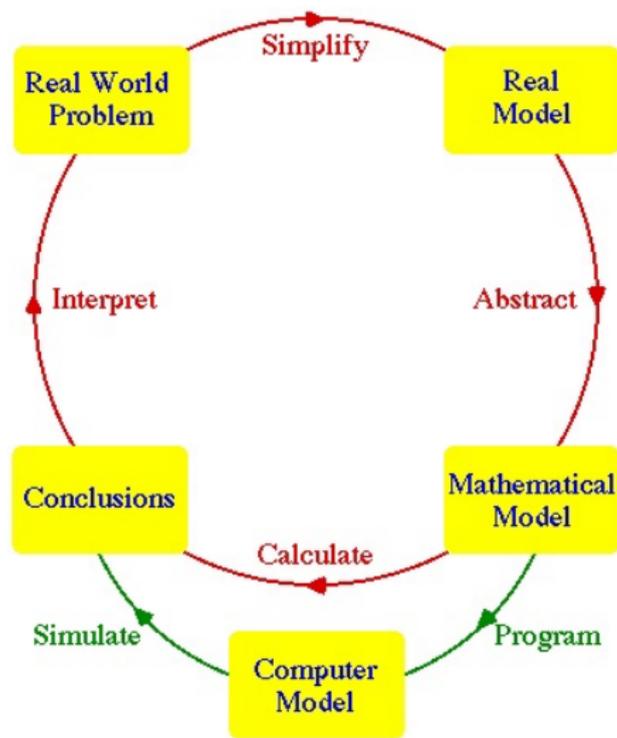
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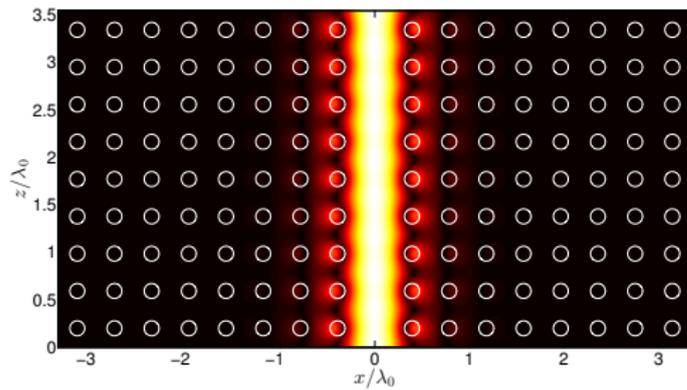
- First half: The structures
- Second half: The methods

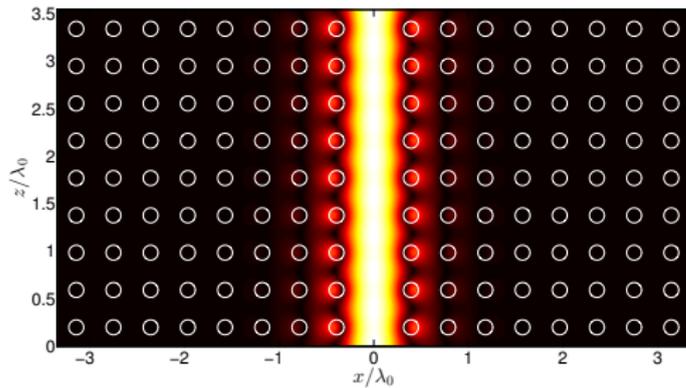
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["Stepping Stones to Mathematical Modeling", Indiana University]



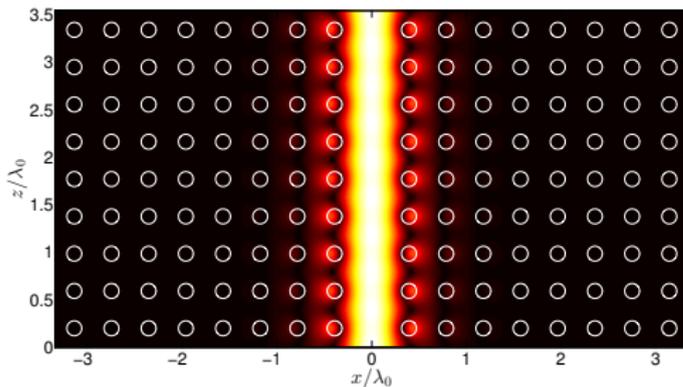


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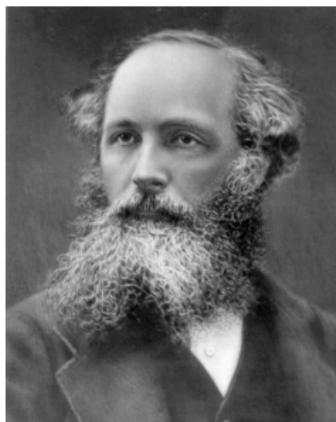


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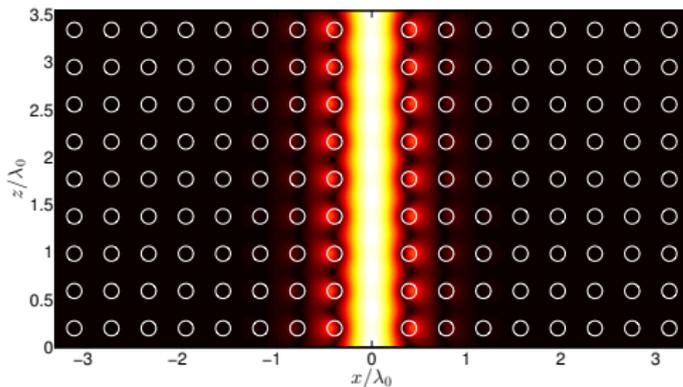


PHILOSOPHICAL
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By J. CLERK MAXWELL, F.R.S.

[Phil. Trans. R. Soc. Lond. **155**, 459-512 (1865)]

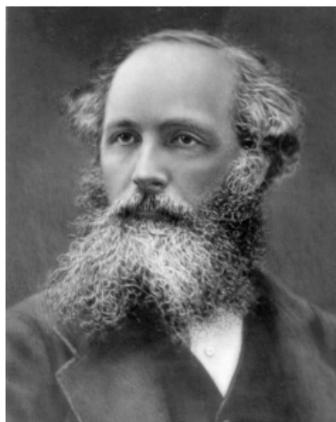


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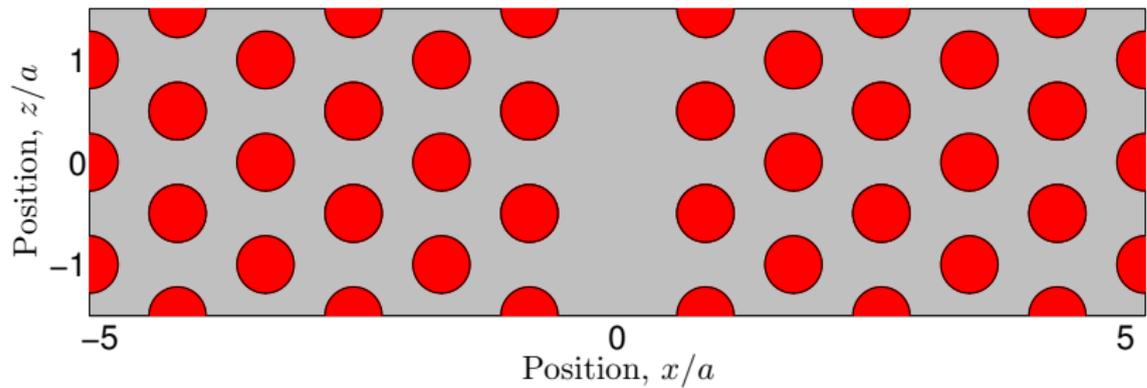
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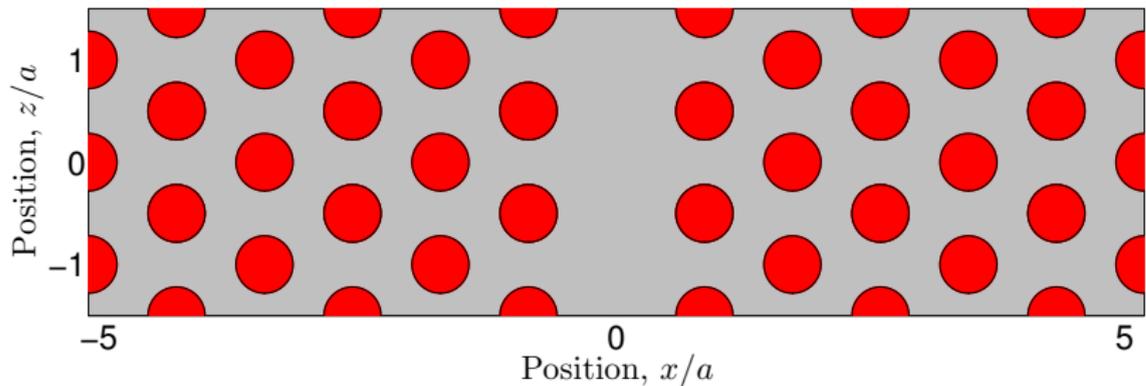
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DTU Indefra

Maxwells ligninger - sværere
end kvantemekanik!

Af Jakob Rosenkrantz de Lasson 4. jun 2014 kl. 07:30

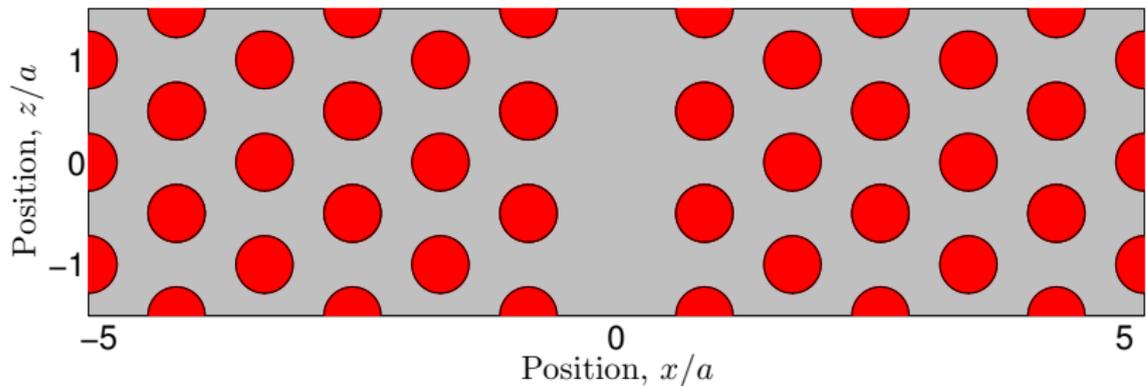




Über die Quantenmechanik der Elektronen in Kristallgittern.

Von **Felix Bloch** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 10. August 1928.)

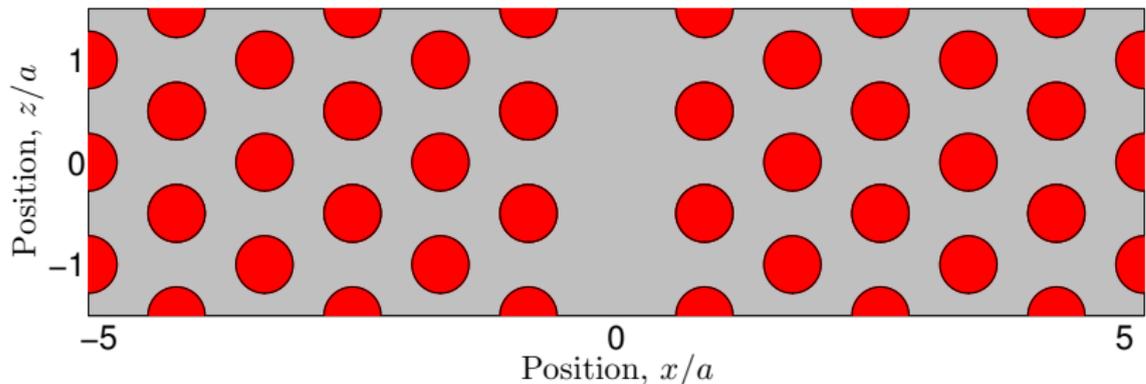


1952 Nobel Prize

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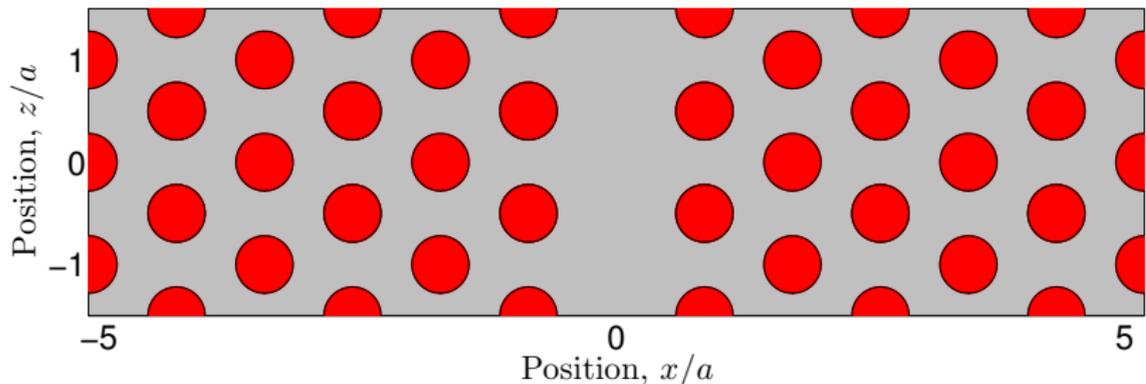
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"By straight Fourier analysis I found to my delight that the wave differed from a plane wave of the free electron only by a periodic modulation"

$$\psi(\mathbf{r}) = u(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})$$

$$u(\mathbf{r} + \mathbf{T}) = u(\mathbf{r})$$



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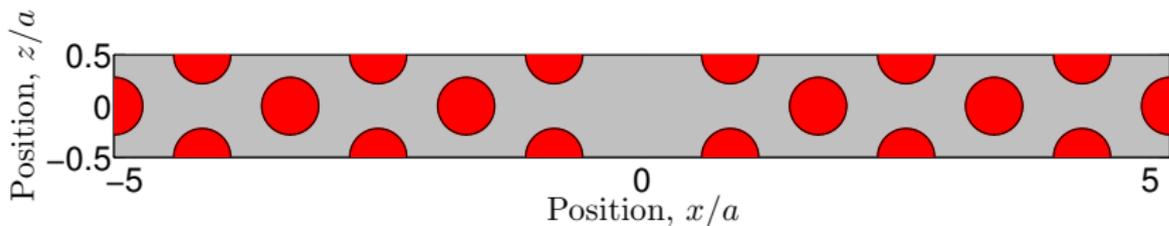
Mit 2 Abbildungen. (Eingegangen am 10. August 1928.)

"By straight Fourier analysis I found to my delight that the wave differed from a plane wave of the free electron only by a periodic modulation"

$$\psi(\mathbf{r}) = u(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})$$

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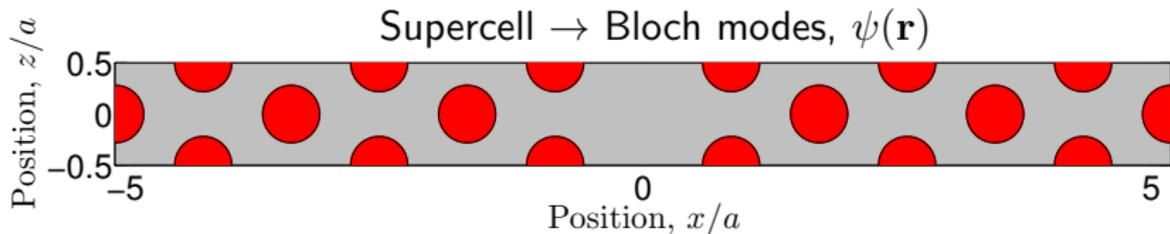
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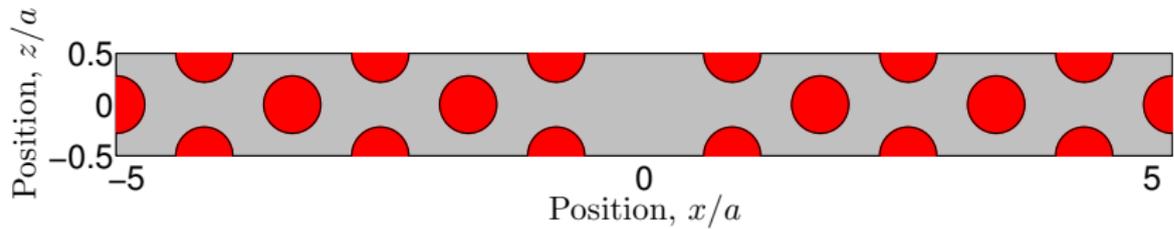
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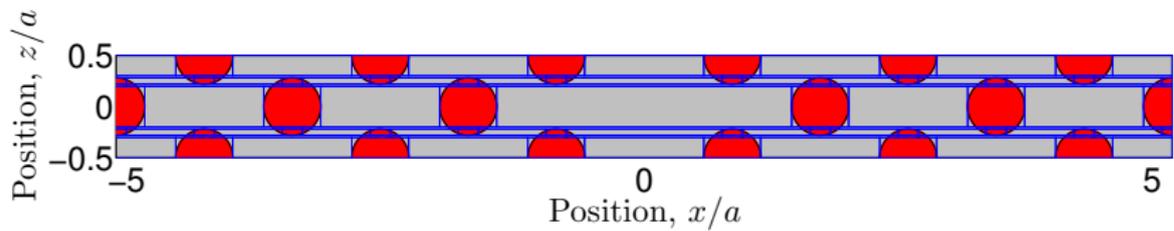
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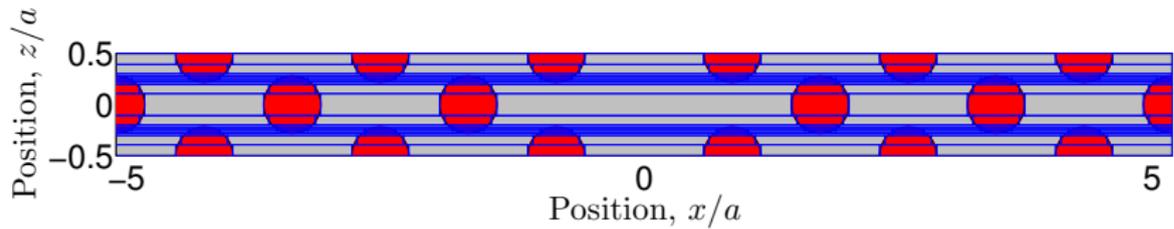
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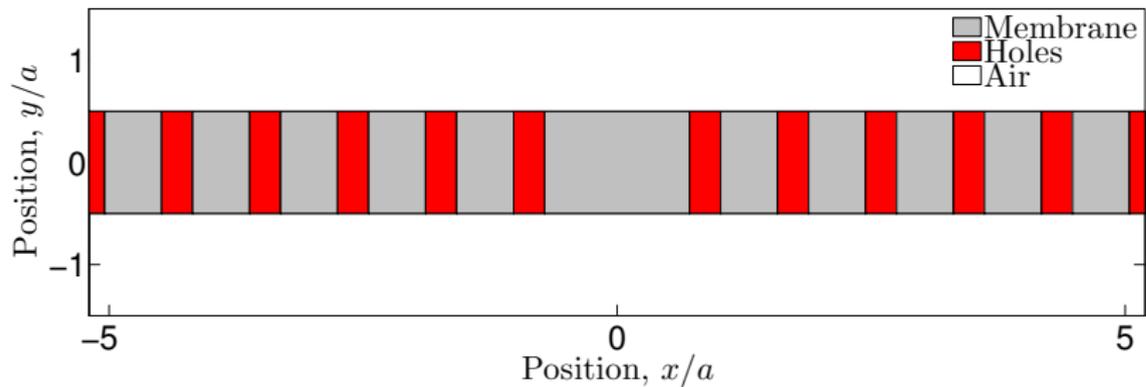
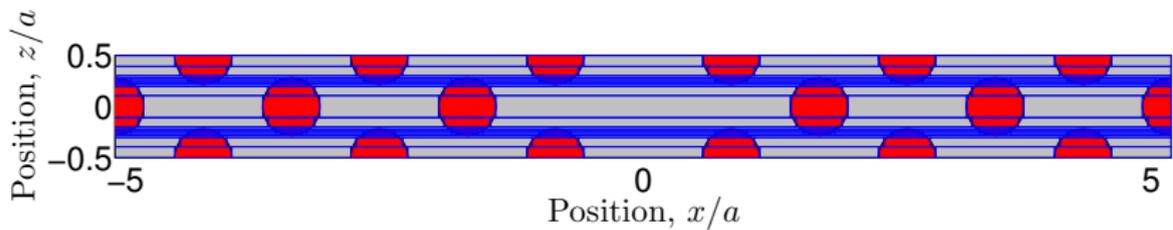
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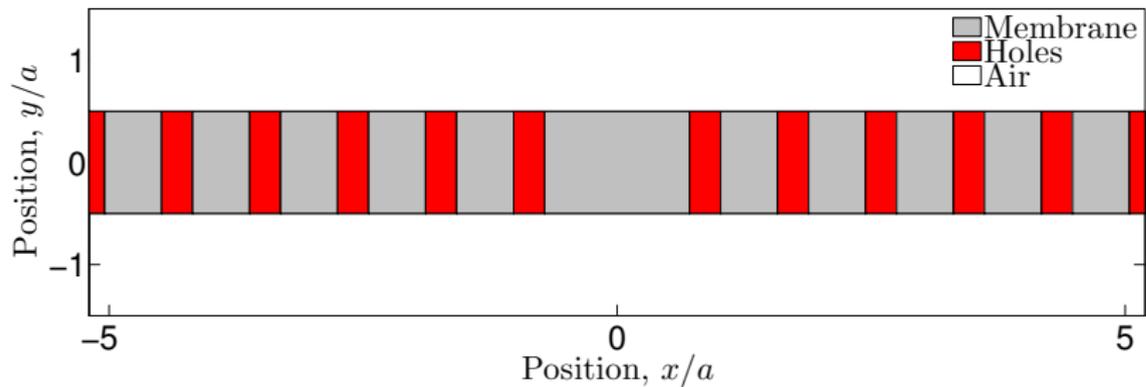
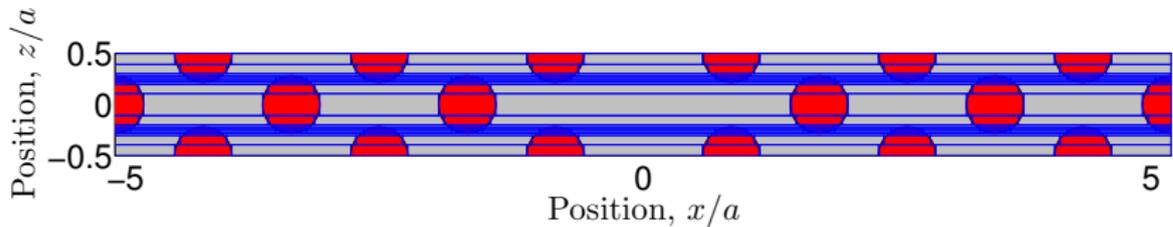
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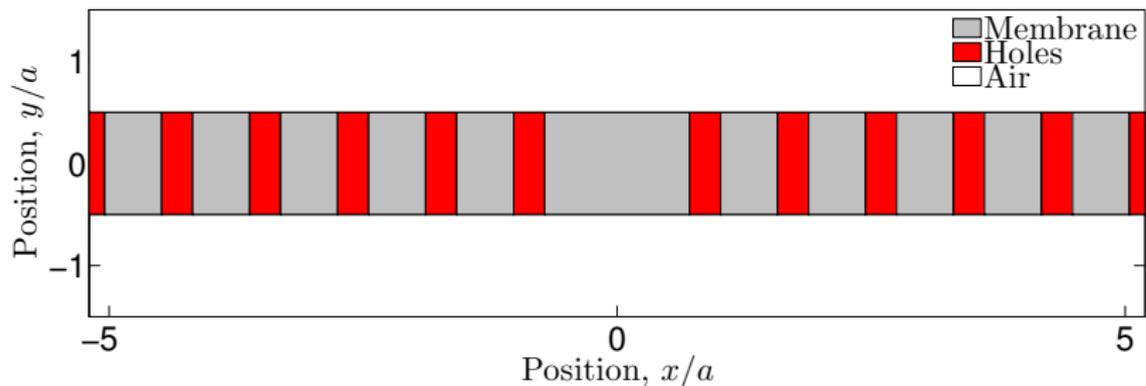
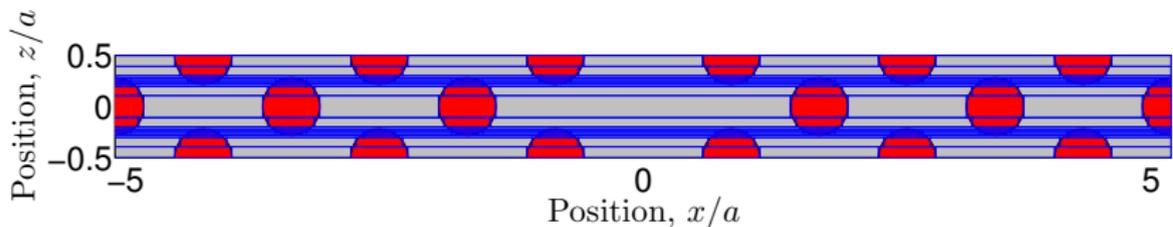








$$+ \mathbf{E}(\mathbf{r}_\perp) \sim \sum_\nu c_\nu \exp(i\mathbf{k}_\nu \cdot \mathbf{r}_\perp)$$



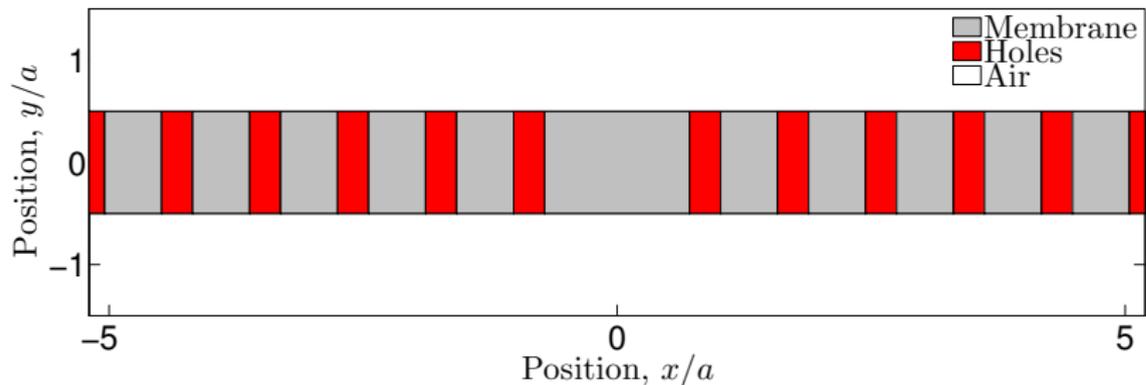
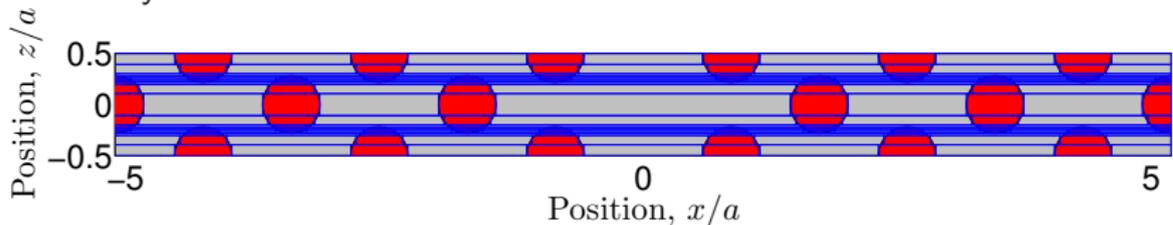
$$+ \mathbf{E}(\mathbf{r}_\perp) \sim \sum_\nu c_\nu \exp(i\mathbf{k}_\nu \cdot \mathbf{r}_\perp) \Rightarrow$$



$$\mathbf{M}d = \lambda \mathbf{N}d$$

$$\lambda = \exp(ik_z a)$$

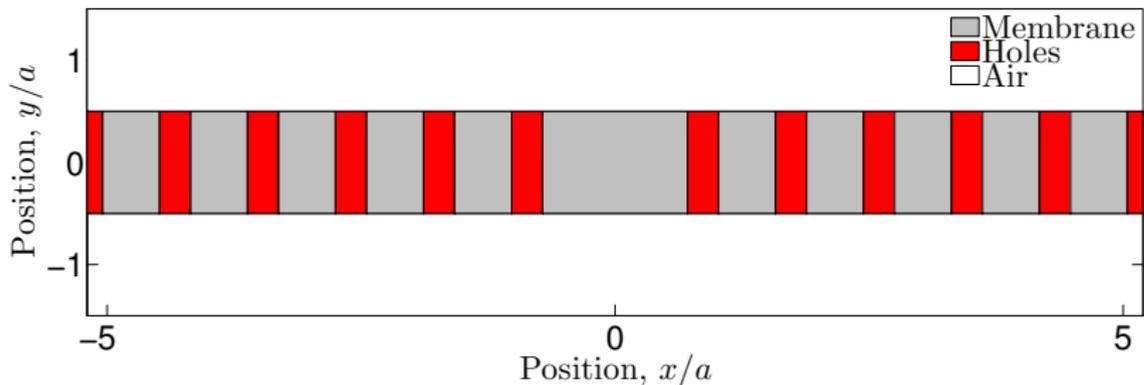
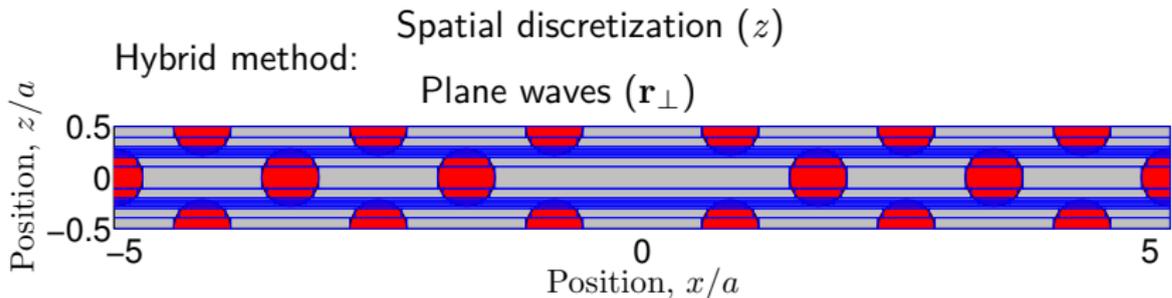
Hybrid method:



$$+ \mathbf{E}(\mathbf{r}_\perp) \sim \sum_\nu c_\nu \exp(i\mathbf{k}_\nu \cdot \mathbf{r}_\perp) \Rightarrow$$



$$\begin{aligned} \mathbf{M}d &= \lambda \mathbf{N}d \\ \lambda &= \exp(ik_z a) \end{aligned}$$

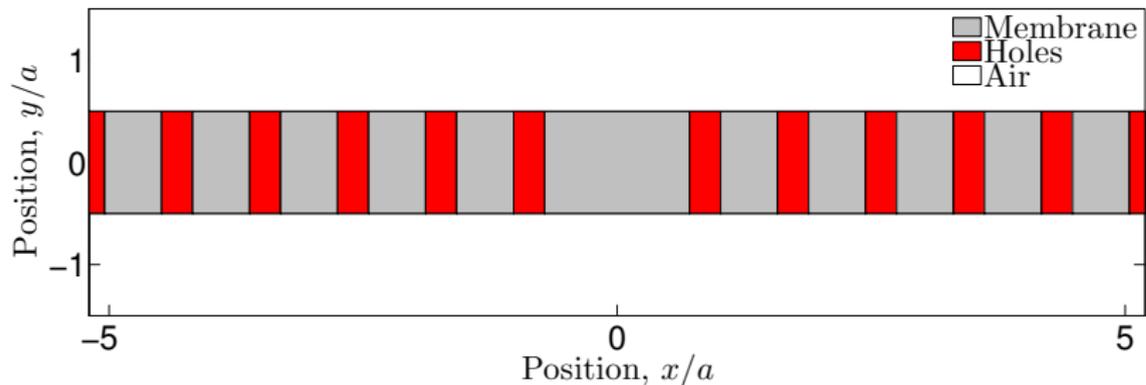
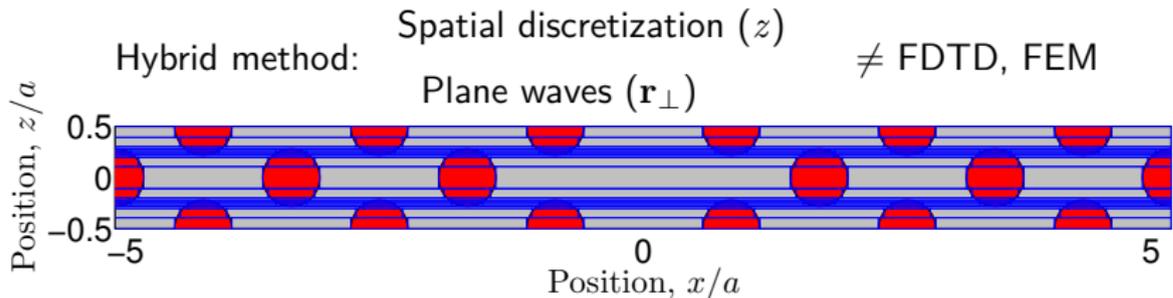


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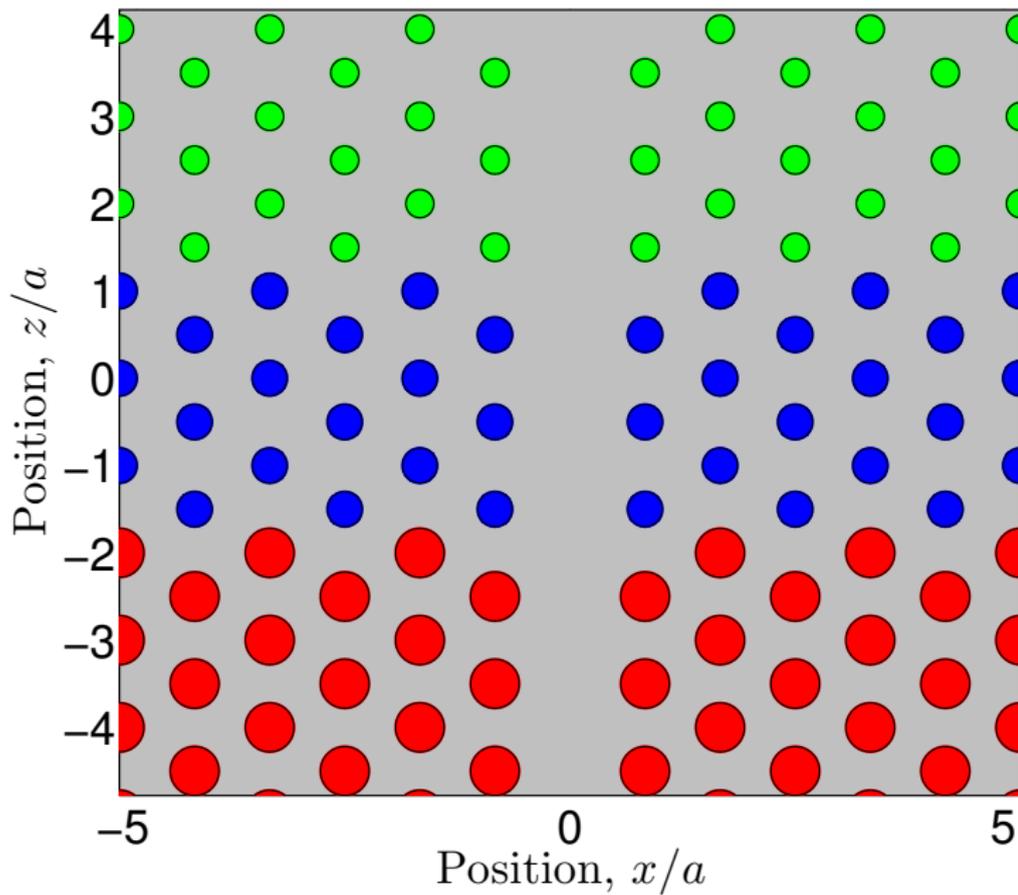


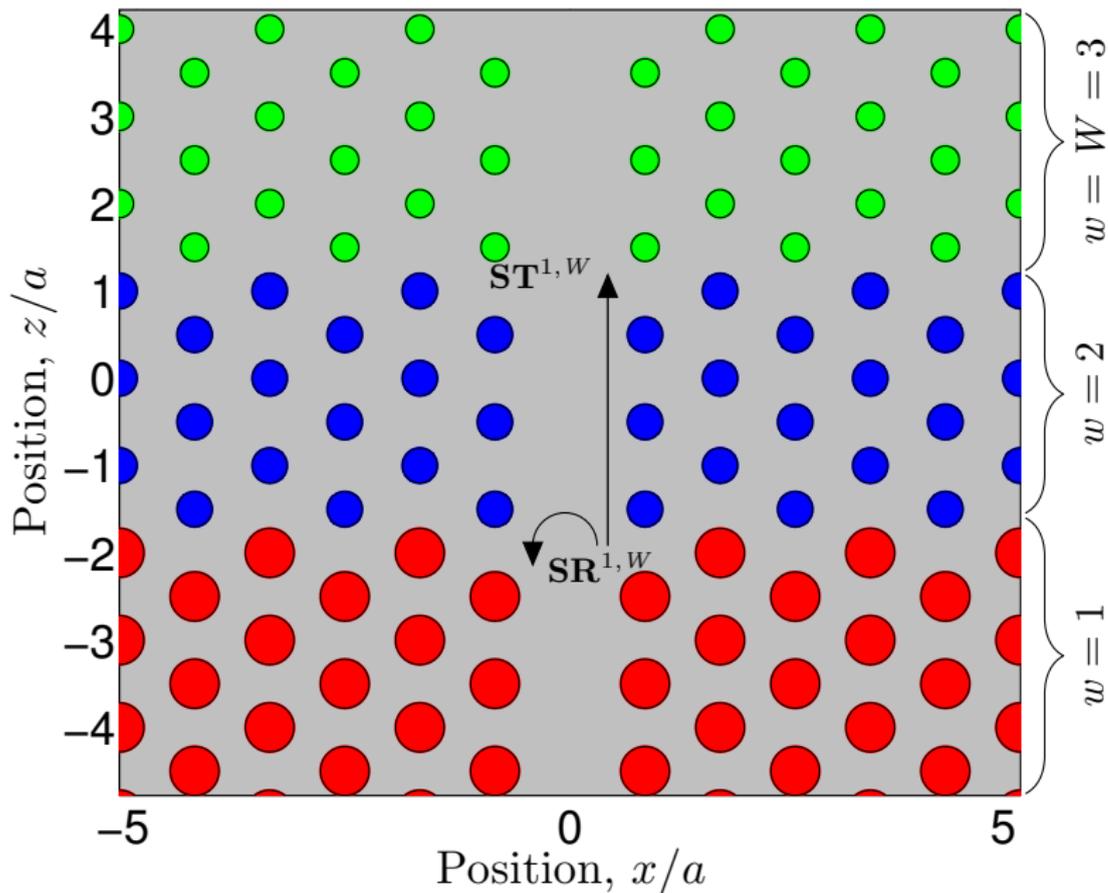
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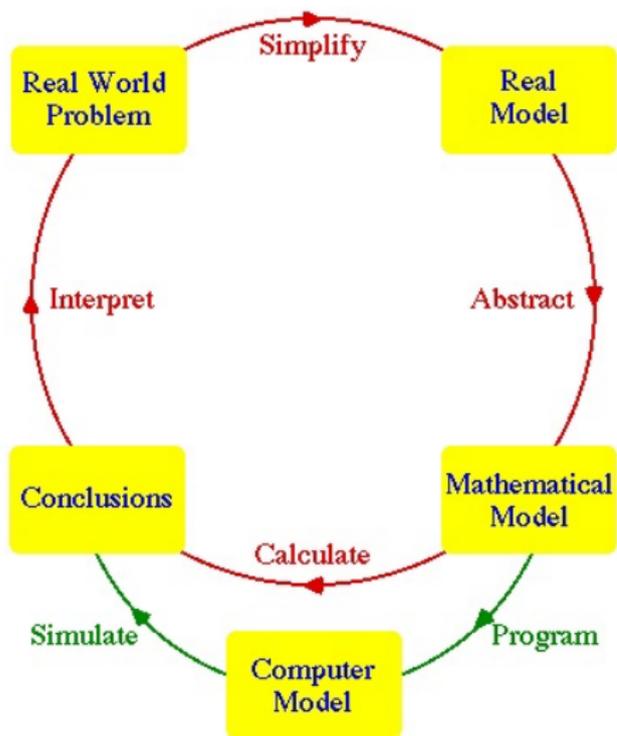


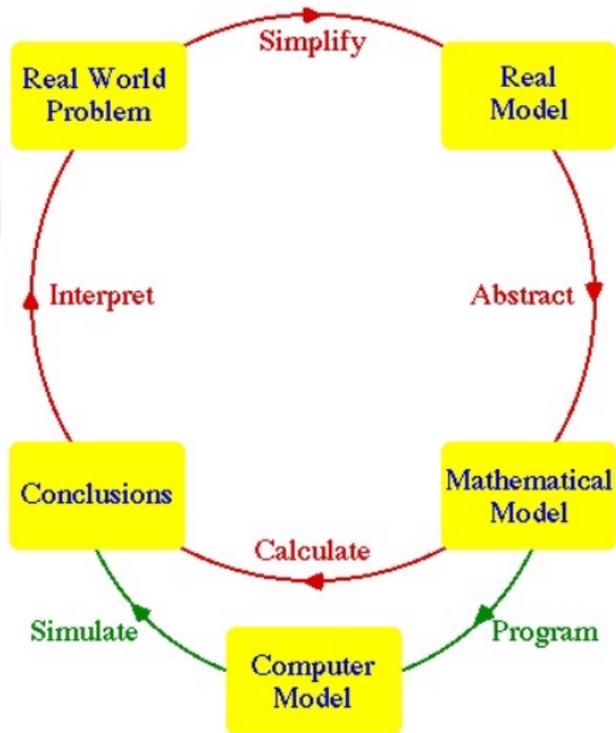
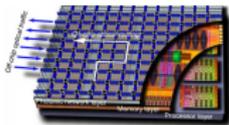
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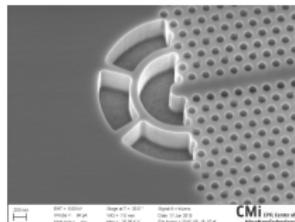
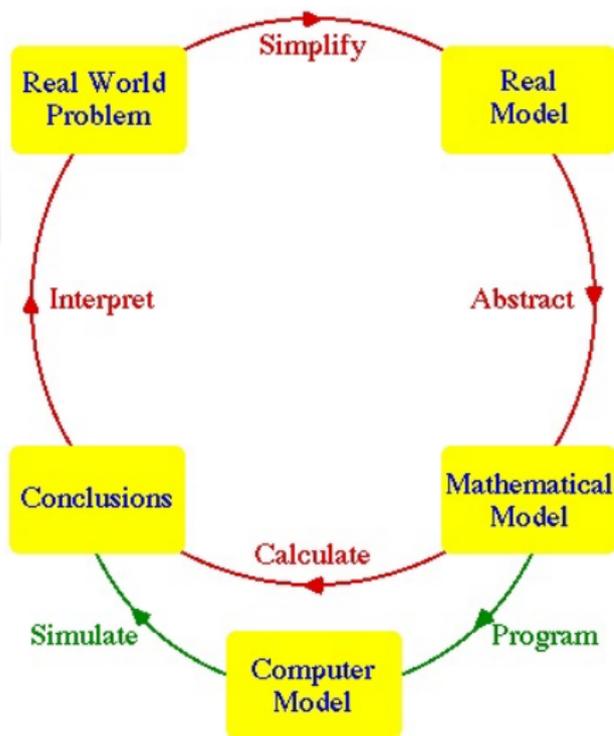
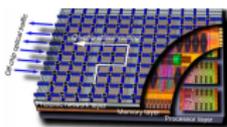
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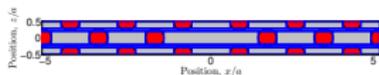
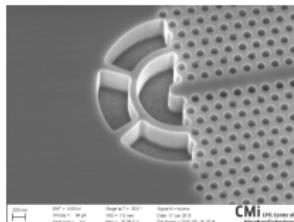
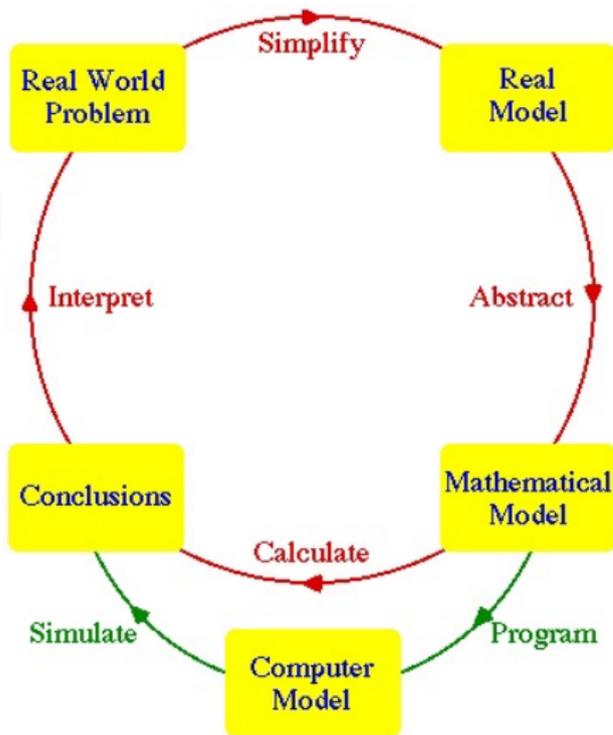
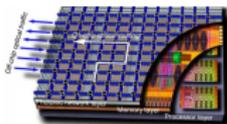








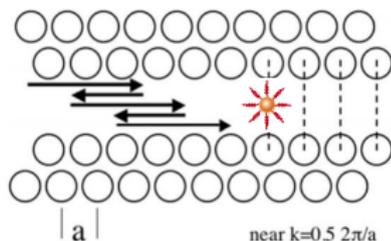




III. *What* do we do in our research?

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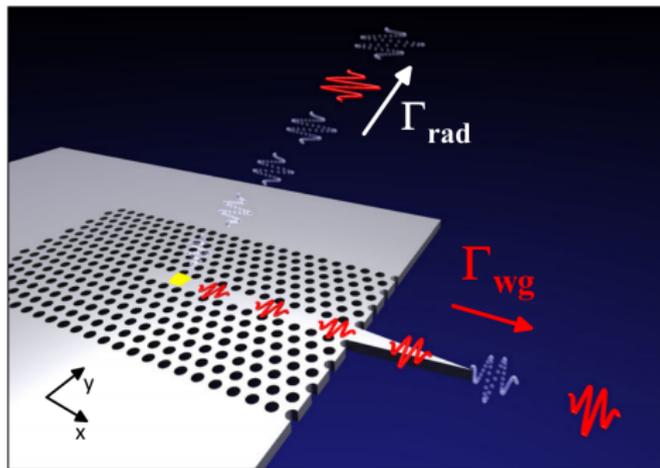
- First half: Light control
with slow light waveguides



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with slow light waveguides





[M. Arcari *et al.*, Phys. Rev. Lett. **113**, 093603 (2014)]



See also: [V. S. C. Manga Rao and S. Hughes, Phys. Rev. B **75**, 205437 (2007)]

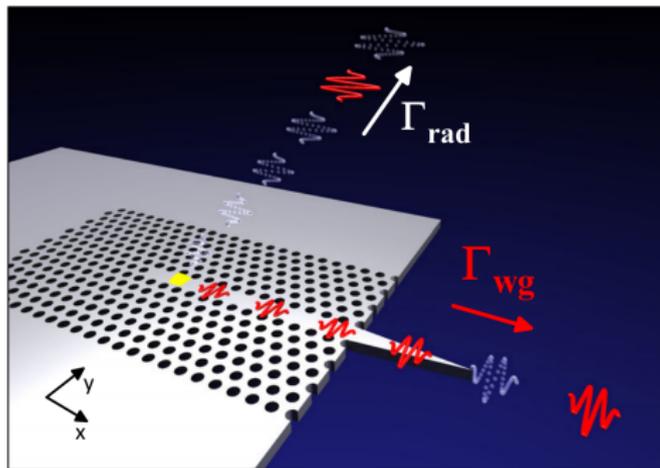
[G. Lecamp *et al.*, Phys. Rev. Lett. **99**, 023902 (2007)]

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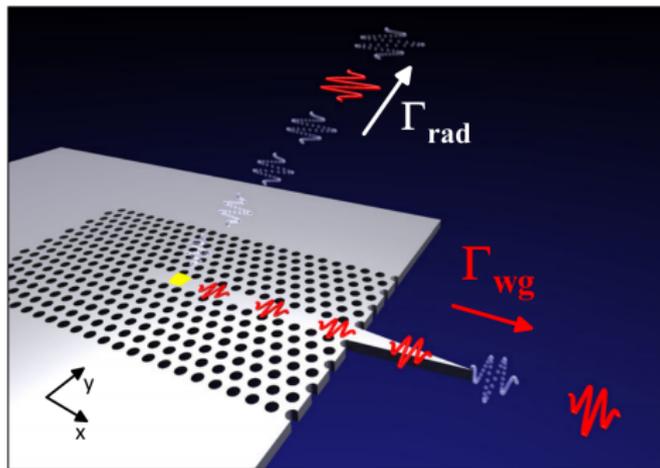
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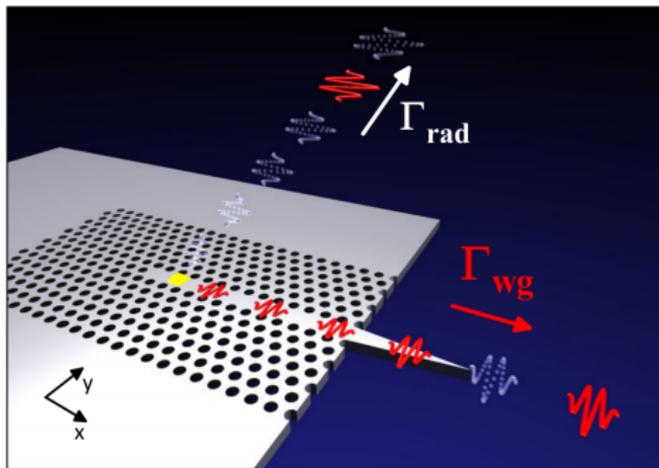
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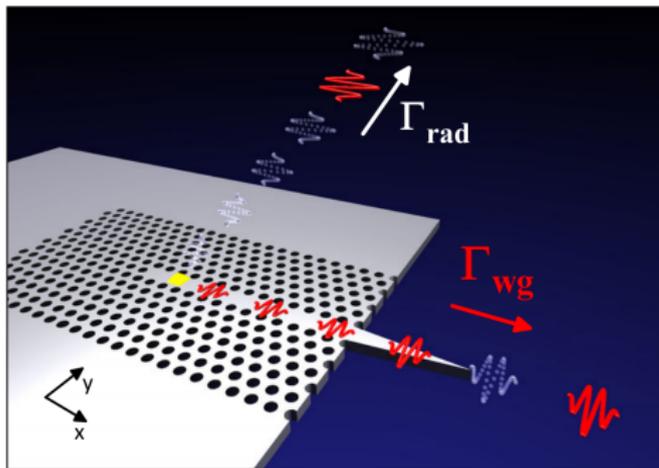
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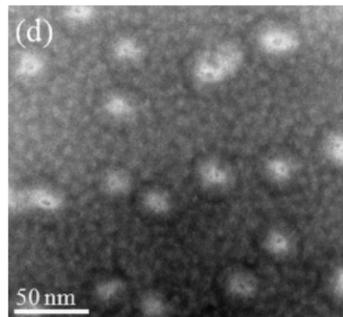
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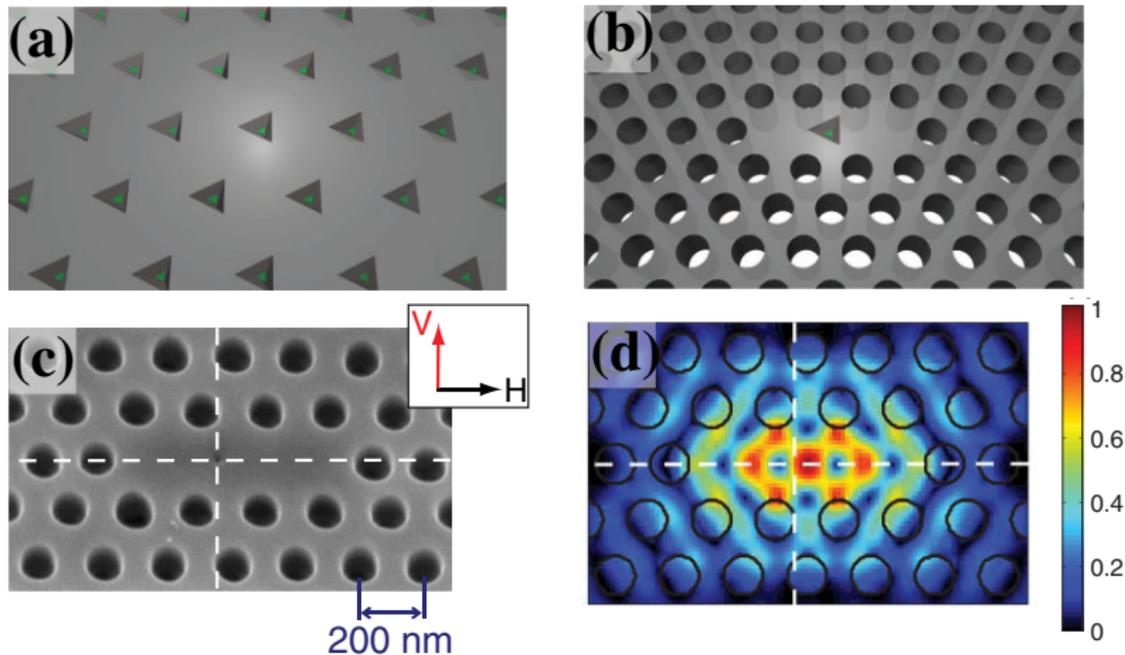
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[T. Ba Hoang *et al.*, Appl. Phys. Lett. **100**, 061122 (2012)]



[E. Semenova *et al.*, Appl. Phys. Lett. **99**, 101106 (2011)]



[M. Calic *et al.*, Phys. Rev. Lett. **106**, 227402 (2011)]

$$\frac{\rho_G}{\rho_{\text{Bulk}}} = \frac{3}{4\pi^2} \left(\frac{\lambda_0}{n}\right)^3 \frac{Q}{V_{\text{eff}}} \eta$$

$$Q = \frac{\omega a}{2c} n_G$$

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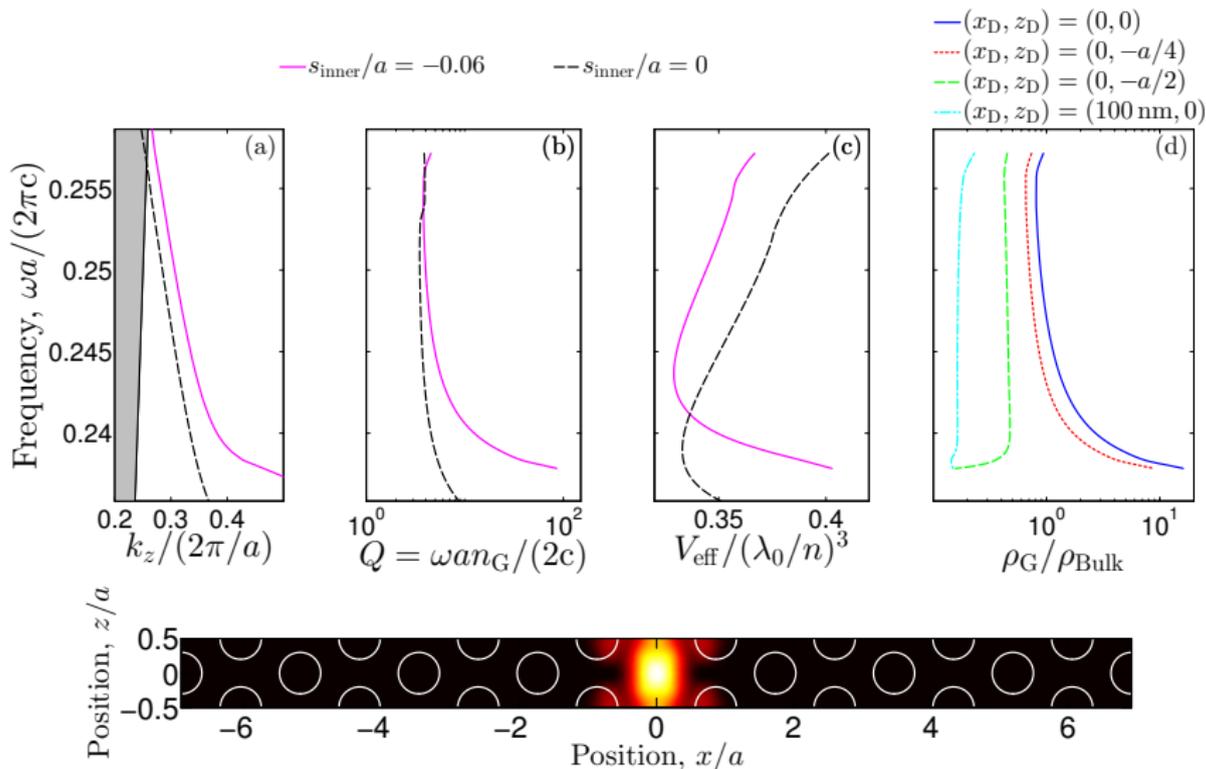
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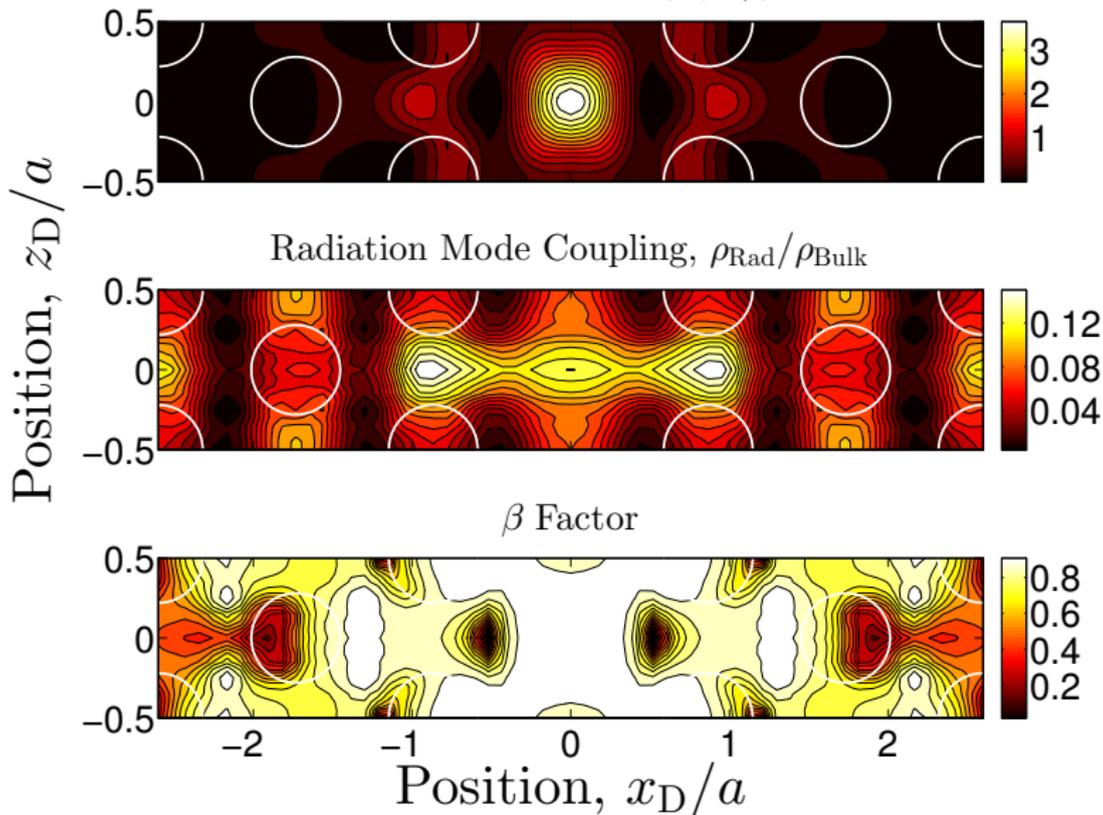
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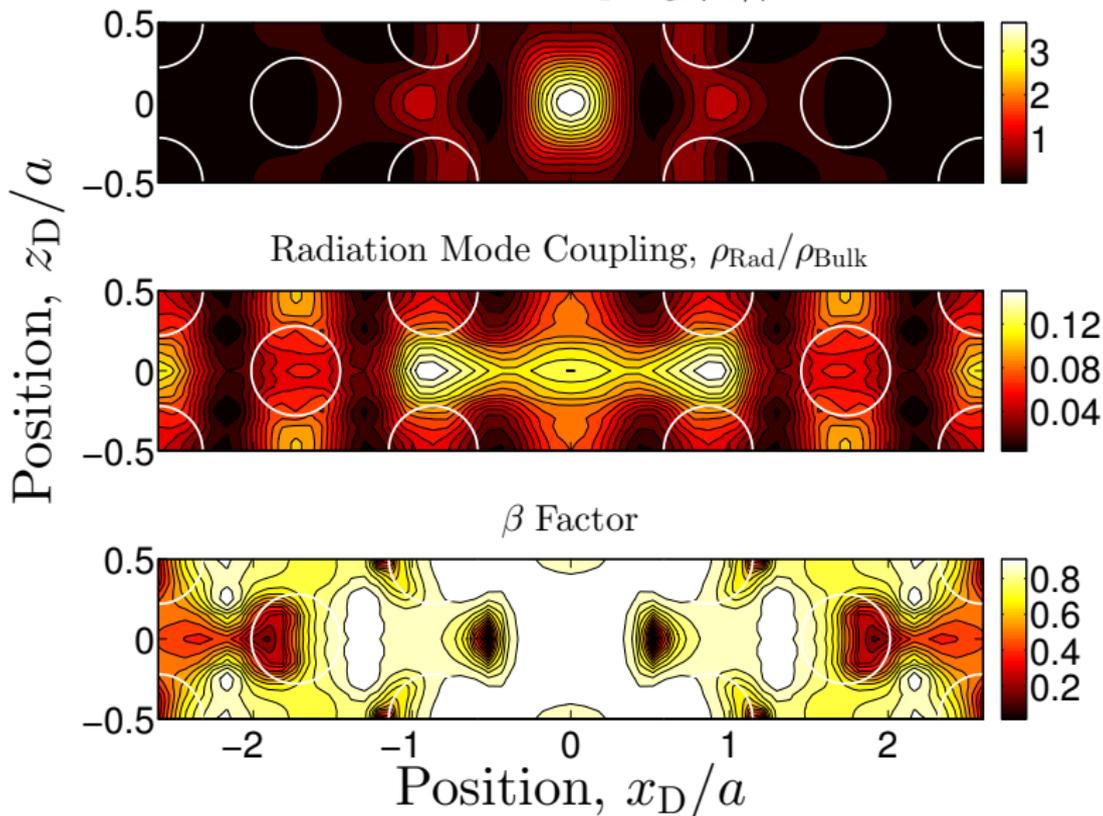
$$n_G = 27$$

Guided Mode Coupling, $\rho_G/\rho_{\text{Bulk}}$

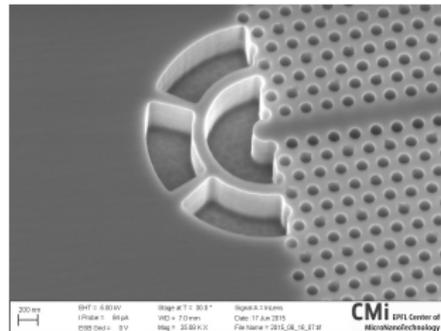


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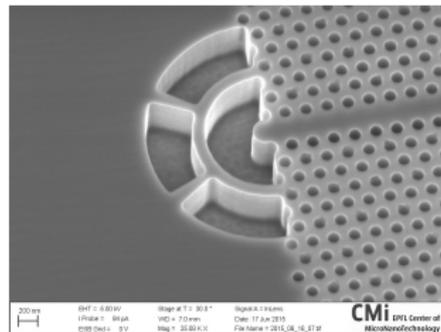
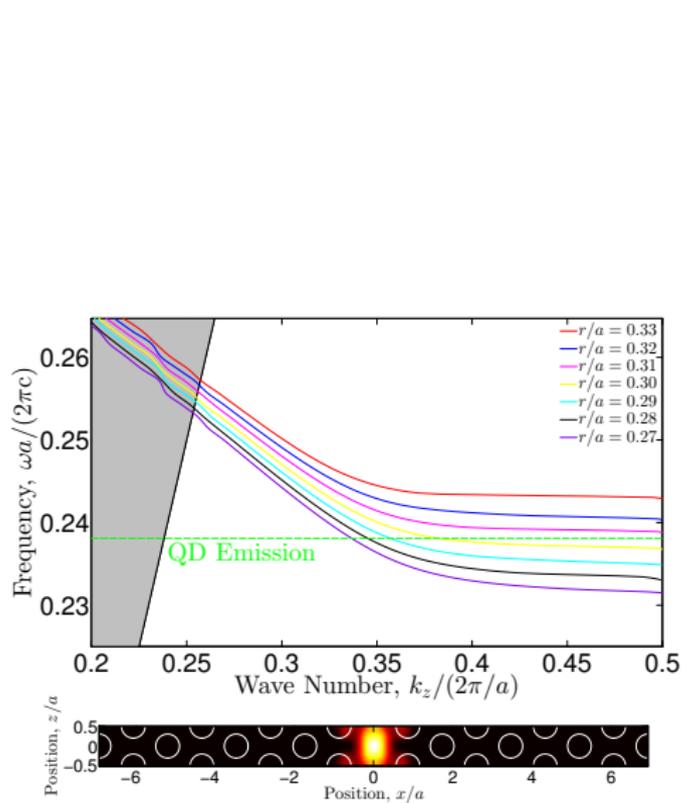
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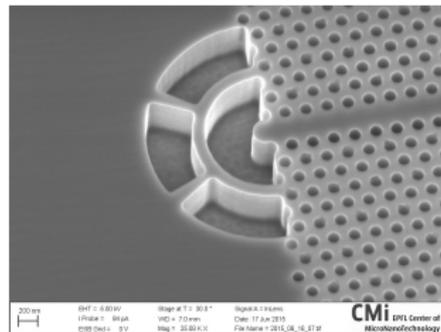
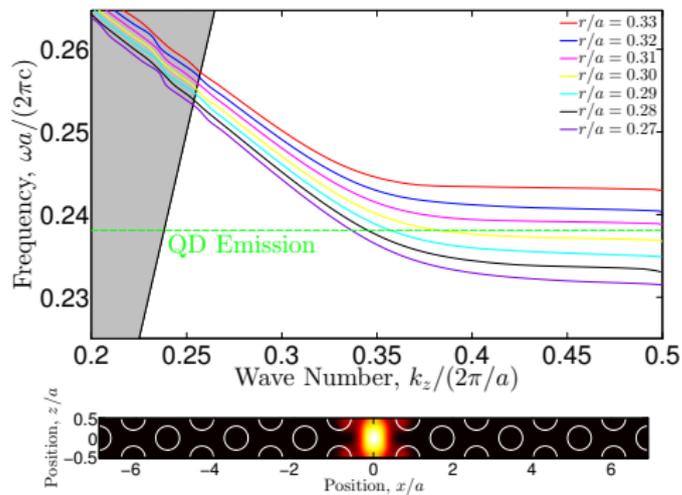
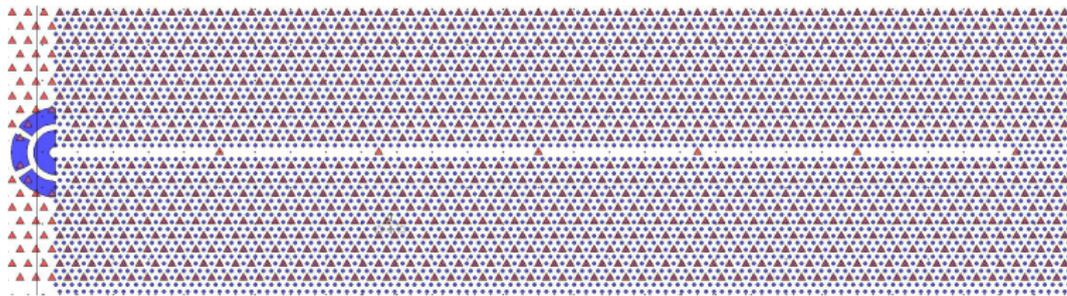
Position matters!



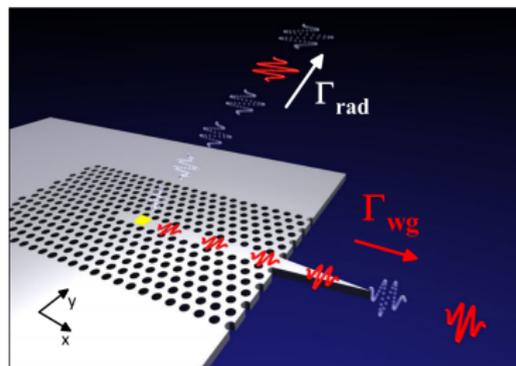
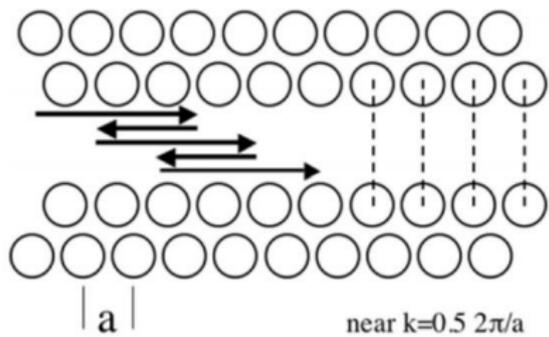
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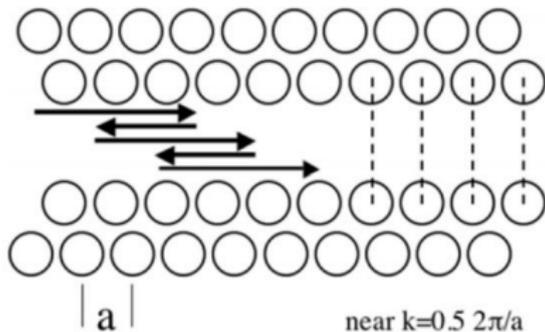


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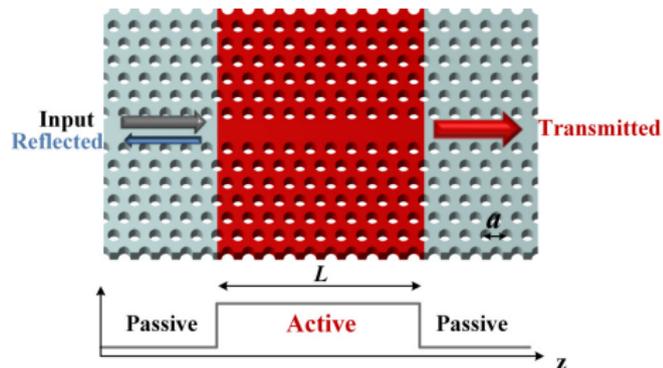
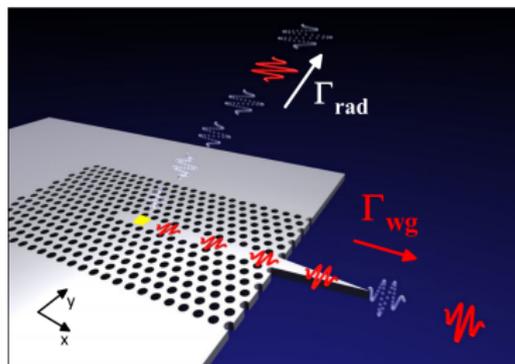


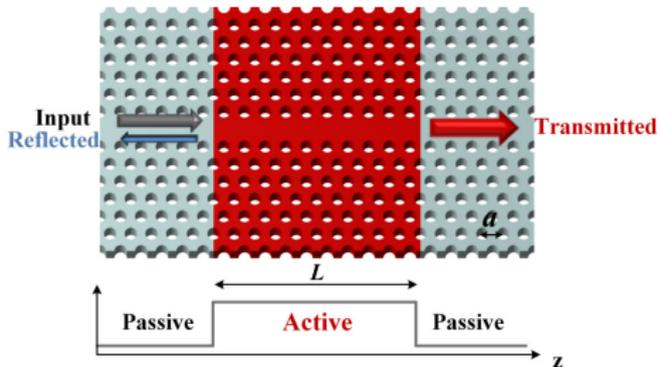
The photonic band edge laser: A new approach to gain enhancement

Jonathan P. Dowling, Michael Scalora, Mark J. Bloemer, and Charles M. Bowden
 Weapons Sciences Directorate, AMSMI-RD-W/S, Research, Development, and Engineering Center,
 U. S. Army Missile Command, Redstone Arsenal, Alabama 35898-5248

(Received 23 September 1993; accepted for publication 8 November 1993)

[J. P. Dowling *et al.*, J. Appl. Phys. **75**, 1896 (1994)]

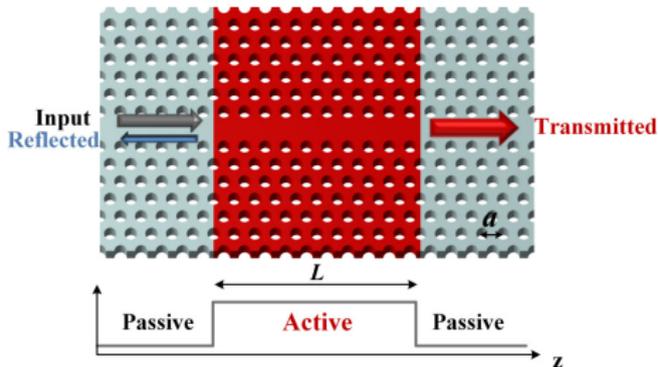




[Y. Chen *et al.*, Phys. Rev. A **92**, 053839 (2015)]

See also: [S. Ek *et al.*, Nat. Commun. **5**, 5039 (2014)]

[J. Grgić *et al.*, Phys. Rev. Lett. **108**, 183903 (2012)]



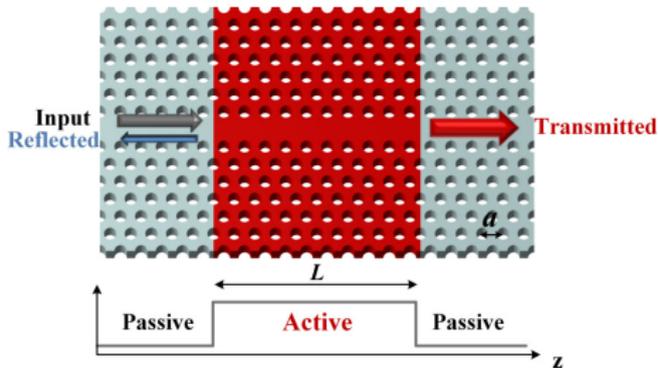
$$\partial_z \psi_+(z) = \frac{i\omega}{c} n_{gz} \chi_{\text{pert}} [\delta(z) \psi_+ + \kappa^*(z) e^{-i2k_z z} \psi_-],$$

$$\partial_z \psi_-(z) = -\frac{i\omega}{c} n_{gz} \chi_{\text{pert}} [\delta(z) \psi_- + \kappa(z) e^{i2k_z z} \psi_+],$$

[Y. Chen *et al.*, Phys. Rev. A **92**, 053839 (2015)]

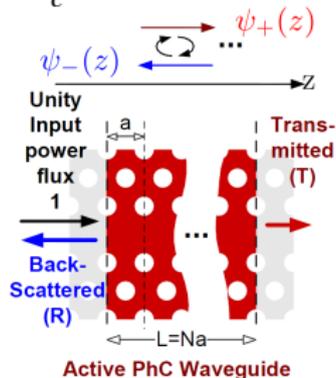
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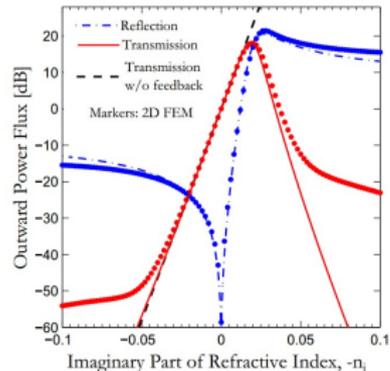
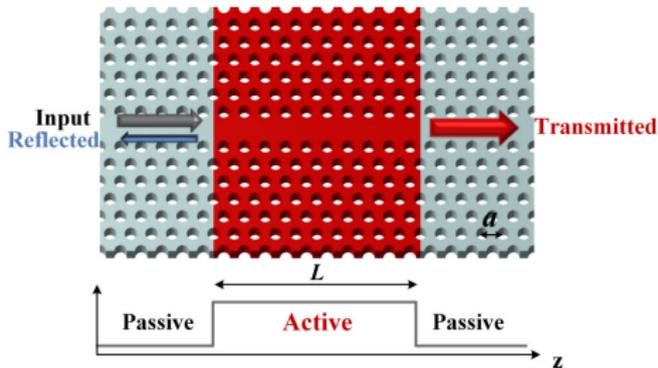
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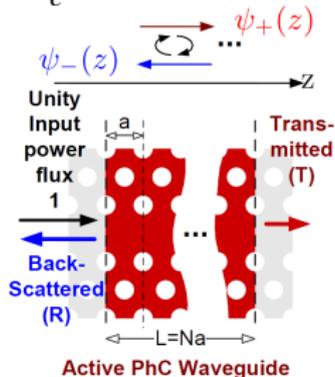
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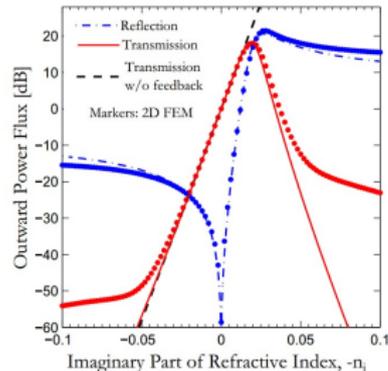
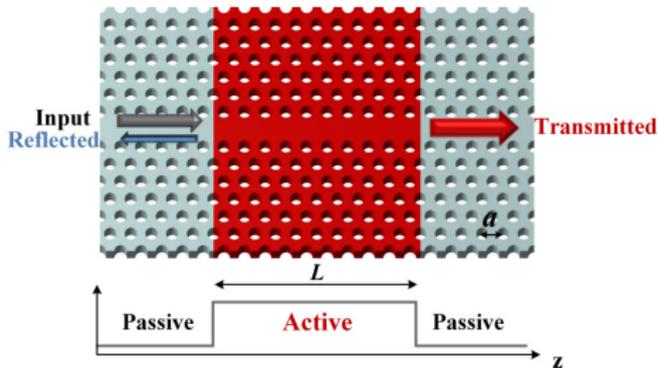
$$\partial_z \psi_-(z) = -\frac{i\omega}{c} n_{gz} \chi_{\text{pert}} [\delta(z) \psi_- + \kappa(z) e^{i2k_z z} \psi_+],$$



[Y. Chen *et al.*, *Phys. Rev. A* **92**, 053839 (2015)]

See also: [S. Ek *et al.*, *Nat. Commun.* **5**, 5039 (2014)]

[J. Grgić *et al.*, *Phys. Rev. Lett.* **108**, 183903 (2012)]

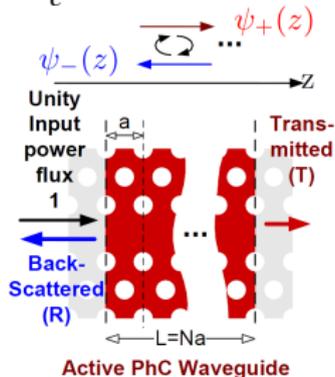


Fundamental Limitations to Gain Enhancement in Periodic Media and Waveguides

Jure Grgić,¹ Johan Raunkjær Ott,¹ Fengwen Wang,² Ole Sigmund,² Antti-Pekka Jauho,³ Jesper Mørk,¹ and N. Asger Mortensen^{1,*}

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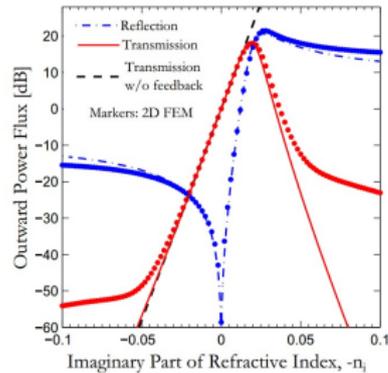
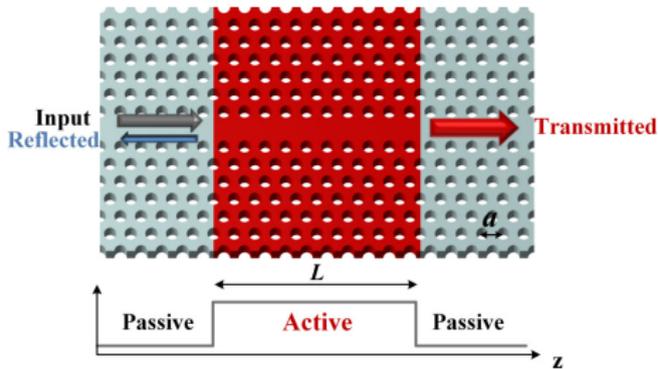
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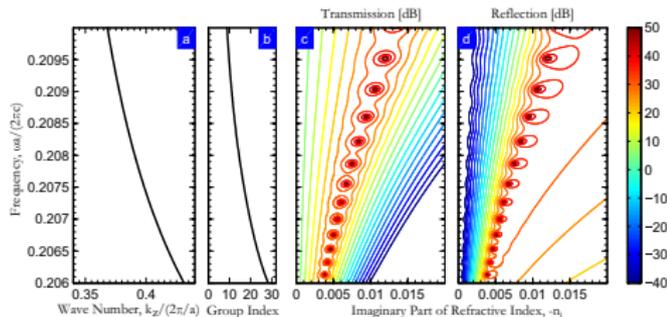
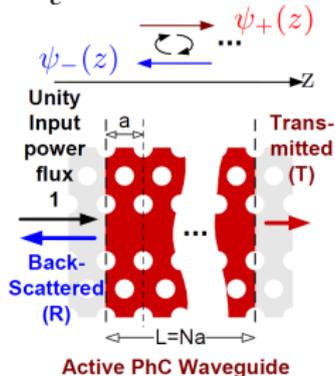


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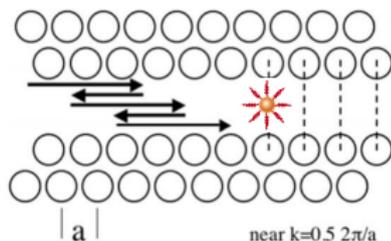
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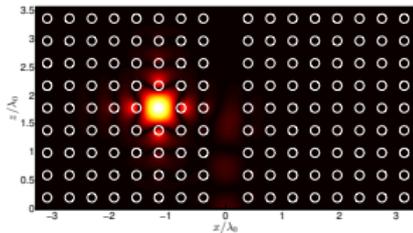


III. *What* do we do in our research?

- First half: Light control
with slow light waveguides

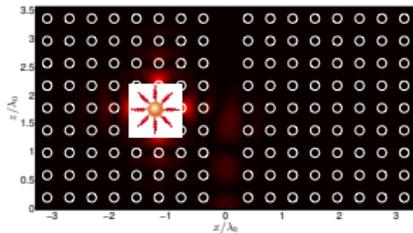
III. *What* do we do in our research?

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B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University*.—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

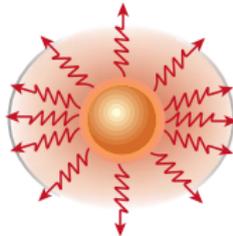
$$A_{\nu} = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2) \text{ sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7 \text{ sec.}^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds! However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V \sim a^3$, and if δ is the skin-depth at frequency ν , $f \sim \lambda^3/a^2\delta$. For a non-resonant circuit $f \sim \lambda^3/a^3$, and for $a < \delta$ it can be shown that $f \sim \lambda^3/a\delta^2$.



... particles, of diameter 10^{-3} cm are mixed in a magnetic medium at room temperature, the emission should establish thermal equilibrium in an order of minutes, for $\nu = 10^7 \text{ sec.}^{-1}$.

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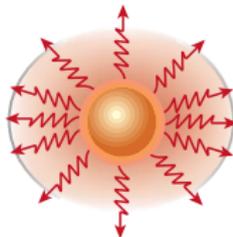
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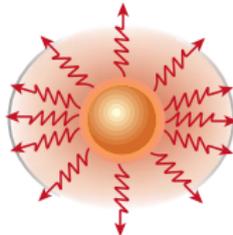
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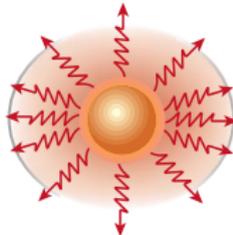
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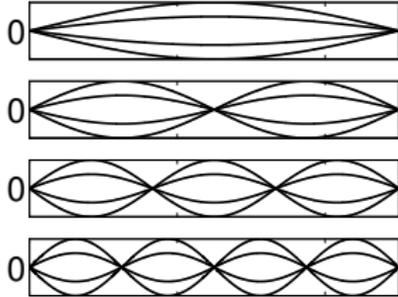


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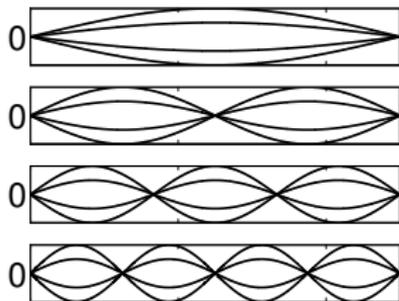
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Quasi-normal modes

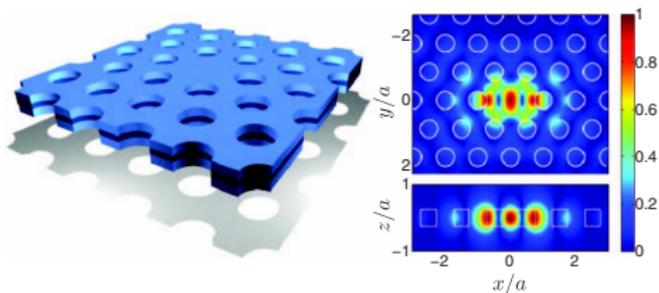
Closed cavity: Normal modes



Closed cavity: Normal modes

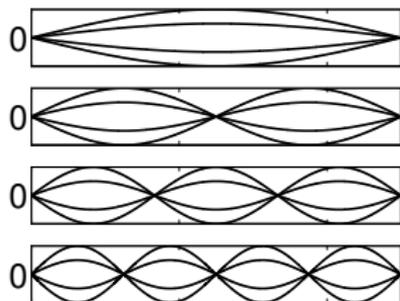


Open cavity: Quasi-normal modes

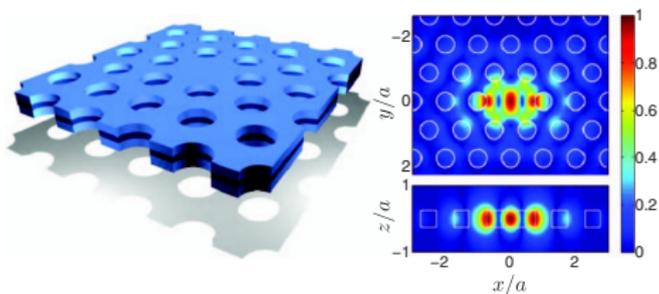


[P. T. Kristensen *et al.*, *Opt. Lett.* **37**, 1649 (2012)]

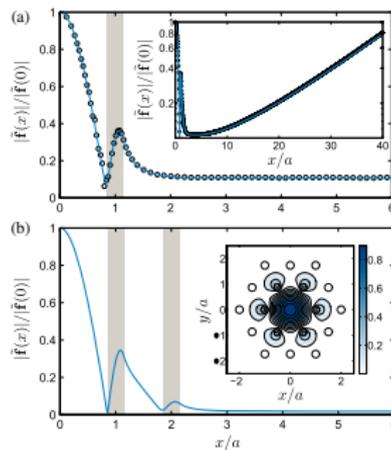
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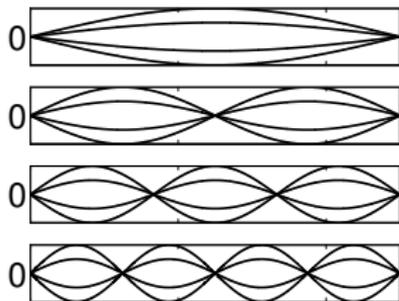
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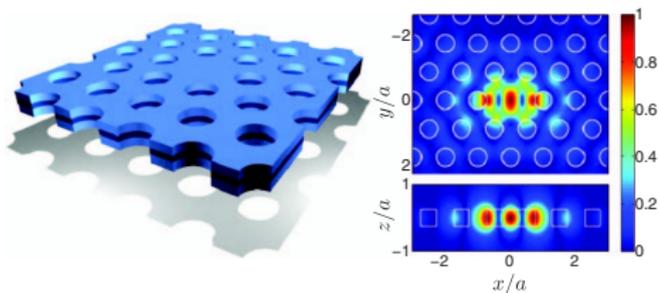
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Closed cavity: Normal modes

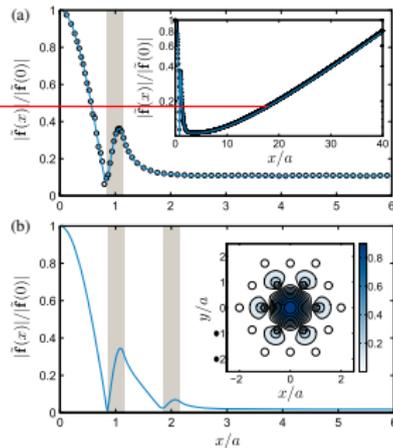


Open cavity: Quasi-normal modes

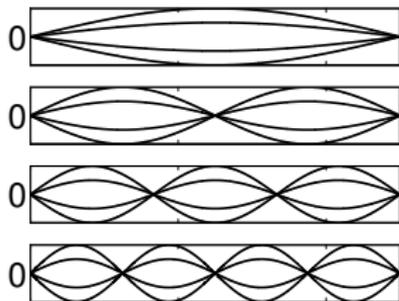


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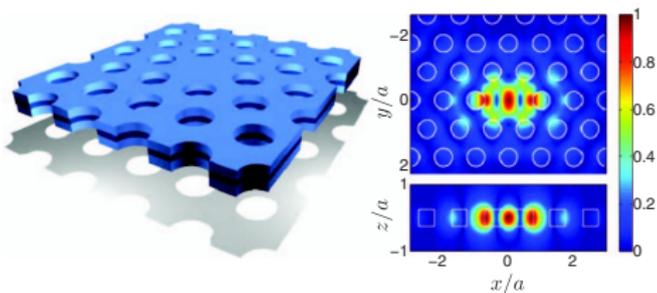
FDTD
and
VIE



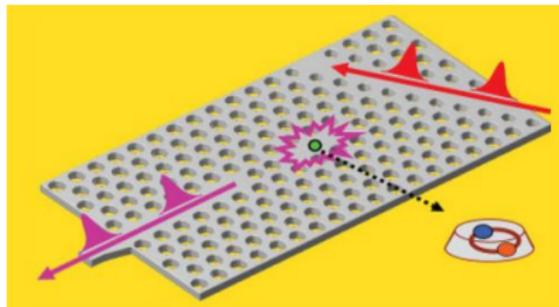
Closed cavity: Normal modes



Open cavity: Quasi-normal modes

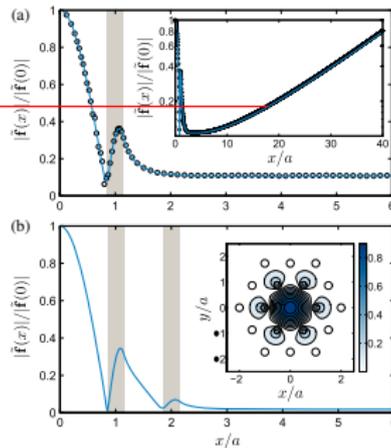


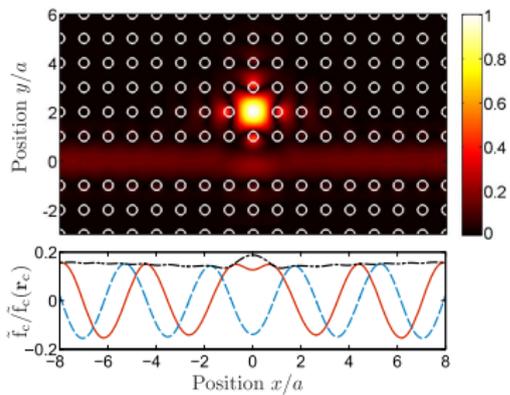
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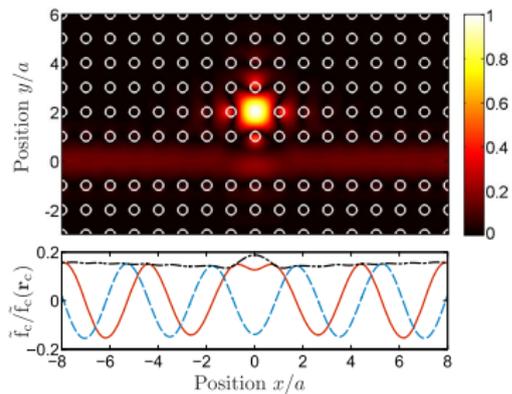
[O. Yao *et al.*, *Laser Photonics Rev.* **4**, 499 (2010)]

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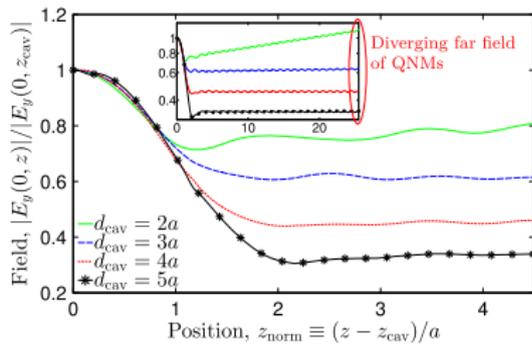




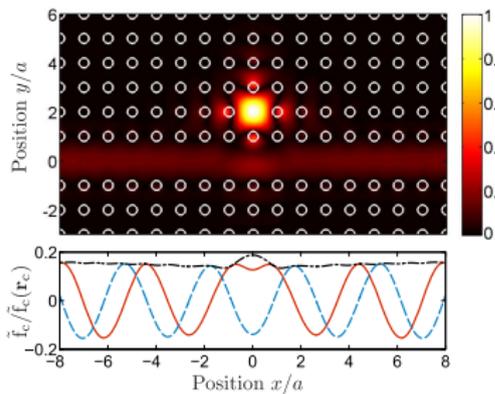
[P. T. Kristensen *et al.*, *Opt. Lett.* **39**, 6359 (2014)]



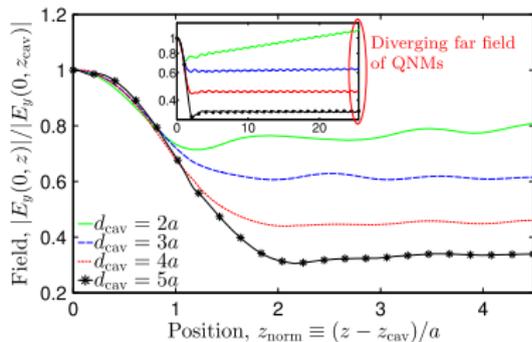
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[J. R. de Lasson *et al.*, *J. Opt. Soc. Am. A* **31**, 2142 (2014)]

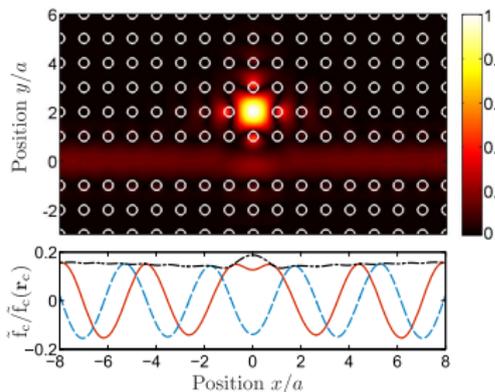


$$\{\mathbf{E}_\mu; \tilde{\omega}_\mu\}$$



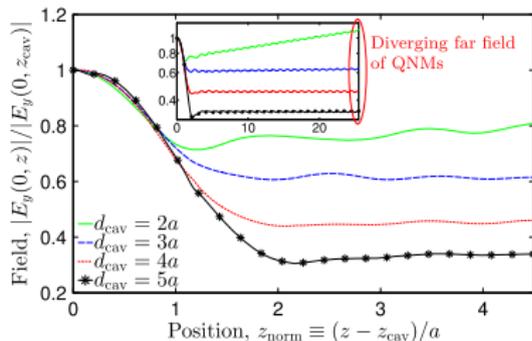
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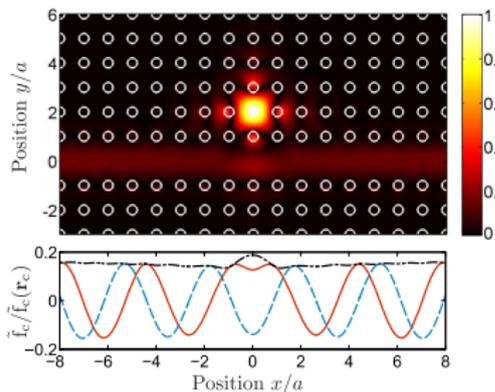
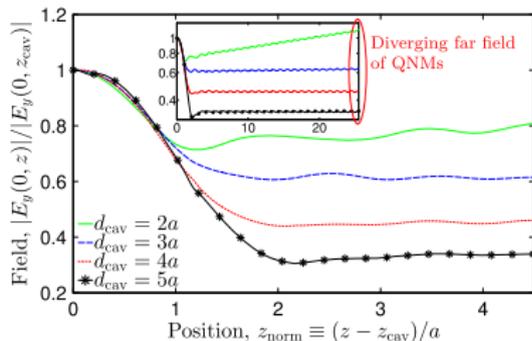


[J. R. de Lasson *et al.*, *J. Opt. Soc. Am. A* **31**, 2142 (2014)]

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LDOS

[L. Novotny and B. Hecht,
"Principles of Nano-Optics" (2012)]


 $\{\mathbf{E}_\mu; \tilde{\omega}_\mu\}$


[P. T. Kristensen *et al.*, *Opt. Lett.* **39**, 6359 (2014)]

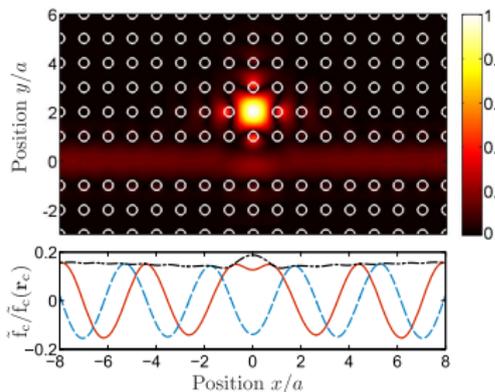
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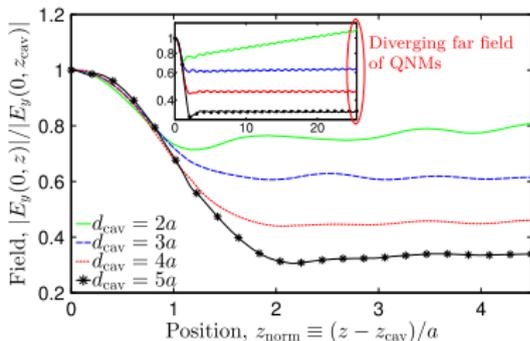
LDOS

$$\rho(\mathbf{r}; \omega) = \frac{2\omega}{\pi c^2} \text{Im} [\hat{\mathbf{n}}_\alpha \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}; \omega) \cdot \hat{\mathbf{n}}_\alpha]$$

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$\{\mathbf{E}_\mu; \tilde{\omega}_\mu\}$



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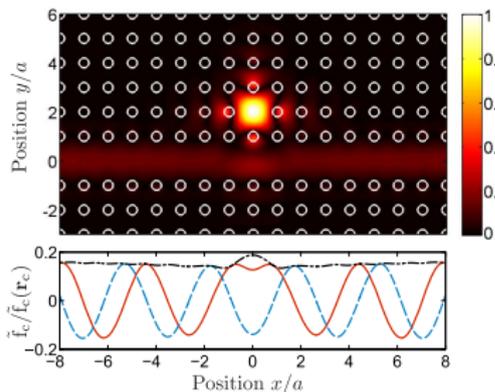
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See also: [C. Sauvan *et al.*, *Phys. Rev. Lett.* **110**, 237401 (2013)]

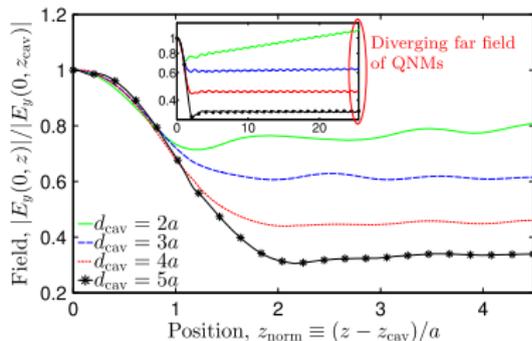
[R.-C. Ge *et al.*, *New J. Phys.* **16**, 113048 (2014)]



$$\{\mathbf{E}_\mu; \tilde{\omega}_\mu\}$$

[P. T. Kristensen *et al.*, *Opt. Lett.* **39**, 6359 (2014)]

[J. R. de Lasson *et al.*, *J. Opt. Soc. Am. A* **31**, 2142 (2014)]



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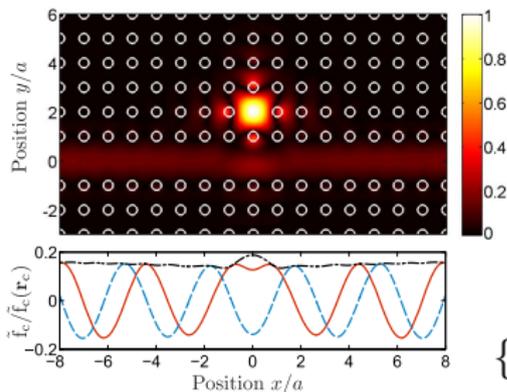
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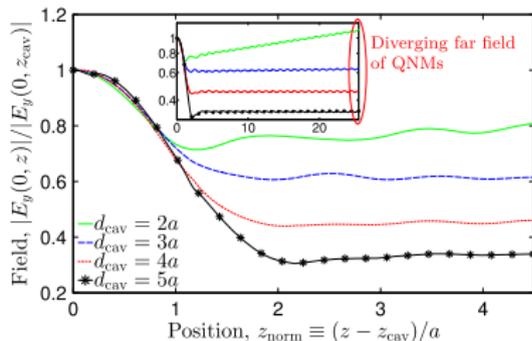
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LDOS

$$\mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega) = \frac{c^2}{2} \sum_{\mu} \frac{\mathbf{E}_\mu(\mathbf{r}) \otimes \mathbf{E}_\mu(\mathbf{r}')}{\tilde{\omega}_\mu(\tilde{\omega}_\mu - \omega)}$$

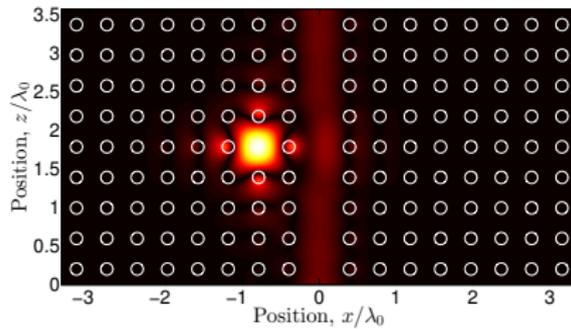
$$\rho(\mathbf{r}; \omega) = \frac{2\omega}{\pi c^2} \text{Im} [\hat{\mathbf{n}}_\alpha \cdot \mathbf{G}(\mathbf{r}, \mathbf{r}; \omega) \cdot \hat{\mathbf{n}}_\alpha]$$

[L. Novotny and B. Hecht,
"Principles of Nano-Optics" (2012)]

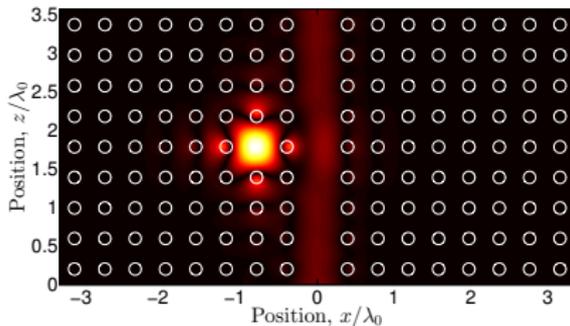
$$\rho(\mathbf{r}; \omega) = \frac{\omega}{\pi} \sum_{\mu} \text{Im} \left[\hat{\mathbf{n}}_\alpha \cdot \frac{\mathbf{E}_\mu(\mathbf{r}) \otimes \mathbf{E}_\mu(\mathbf{r})}{\tilde{\omega}_\mu(\tilde{\omega}_\mu - \omega)} \cdot \hat{\mathbf{n}}_\alpha \right]$$

See also: [C. Sauvan *et al.*, *Phys. Rev. Lett.* **110**, 237401 (2013)]

[R.-C. Ge *et al.*, *New J. Phys.* **16**, 113048 (2014)]



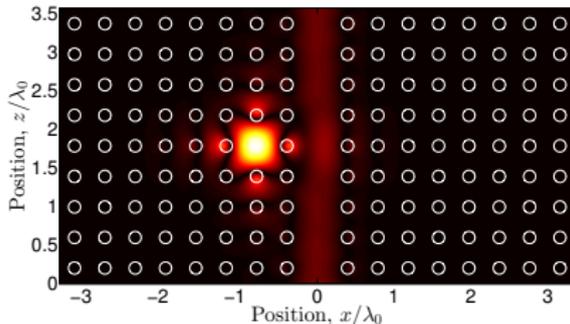
[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]



$$\rho^y(\mathbf{r}_D; \omega) = \frac{\omega}{\pi} \frac{1}{\epsilon(\mathbf{r}_D)} \text{Im} \left[\frac{1}{\tilde{\omega}_\mu(\tilde{\omega}_\mu - \omega)} \frac{1}{a_\mu} \right]$$

$$a_\mu = \frac{1}{\epsilon(\mathbf{r}_D) [\mathbf{E}_\mu(\mathbf{r}_D) \cdot \hat{\mathbf{y}}]^2}$$

[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]

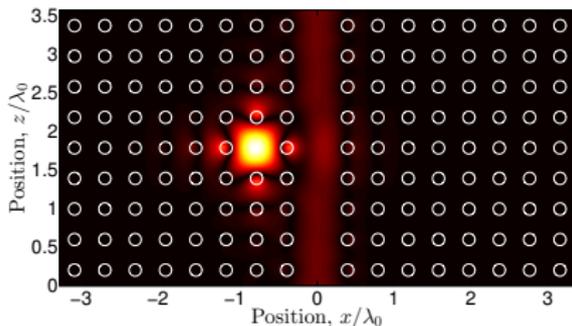


$$\rho^y(\mathbf{r}_D; \omega) = \frac{\omega}{\pi} \frac{1}{\epsilon(\mathbf{r}_D)} \text{Im} \left[\frac{1}{\tilde{\omega}_\mu(\tilde{\omega}_\mu - \omega)} \frac{1}{a_\mu} \right]$$

$$a_\mu = \frac{1}{\epsilon(\mathbf{r}_D) [\mathbf{E}_\mu(\mathbf{r}_D) \cdot \hat{\mathbf{y}}]^2}$$

$$F_P \equiv \frac{\rho^y(\mathbf{r}_D; \omega_\mu)}{\rho_{\text{Bulk}}} \simeq \frac{1}{\pi^2} \left(\frac{\lambda_0}{n(\mathbf{r}_D)} \right)^2 \frac{Q_\mu}{A_\mu}$$

[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]



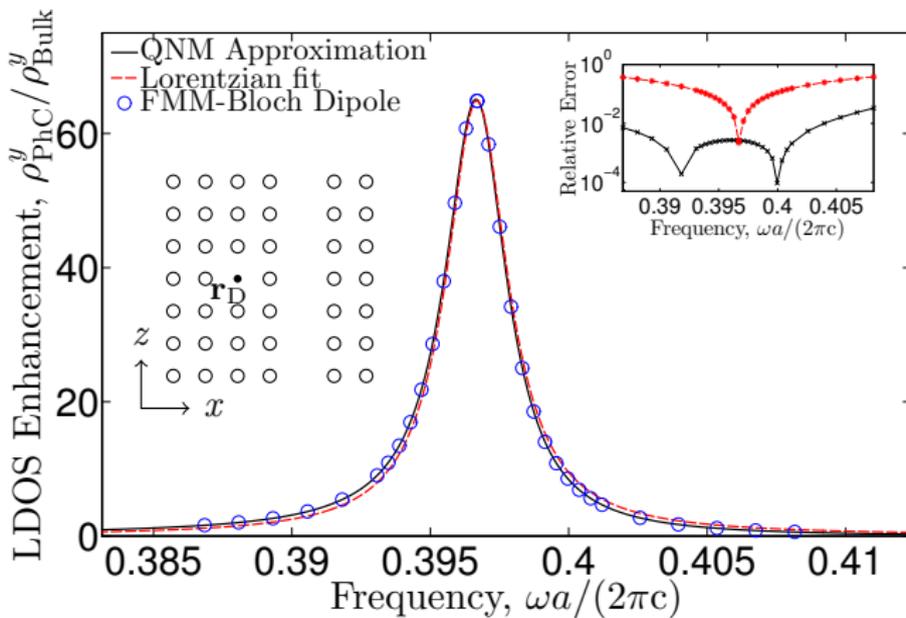
$$\rho^y(\mathbf{r}_D; \omega) = \frac{\omega}{\pi} \frac{1}{\epsilon(\mathbf{r}_D)} \text{Im} \left[\frac{1}{\tilde{\omega}_\mu(\tilde{\omega}_\mu - \omega)} \frac{1}{a_\mu} \right]$$

$$a_\mu = \frac{1}{\epsilon(\mathbf{r}_D) [\mathbf{E}_\mu(\mathbf{r}_D) \cdot \hat{\mathbf{y}}]^2}$$

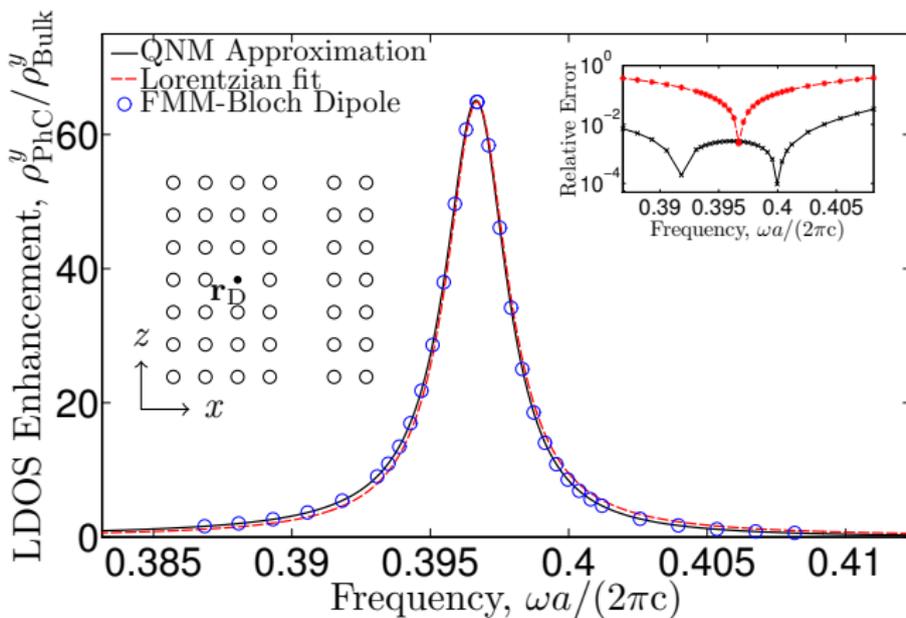
$$F_P \equiv \frac{\rho^y(\mathbf{r}_D; \omega_\mu)}{\rho_{\text{Bulk}}} \simeq \frac{1}{\pi^2} \left(\frac{\lambda_0}{n(\mathbf{r}_D)} \right)^2 \frac{Q_\mu}{A_\mu}$$



[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]

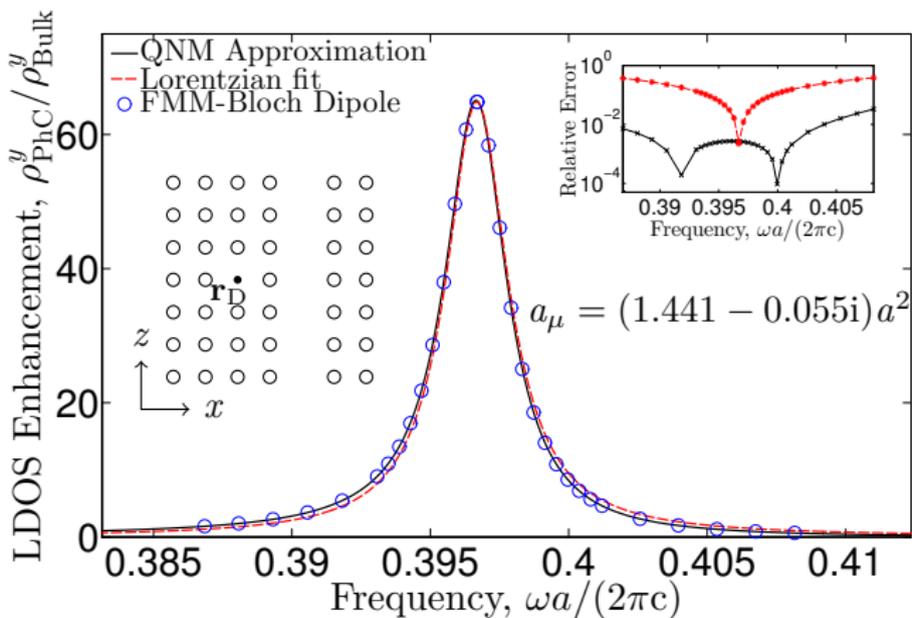


[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]



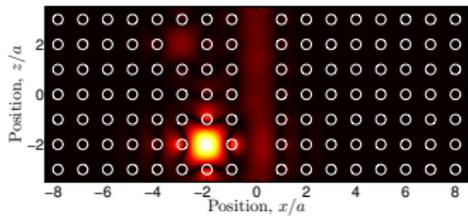
$$\rho_{\text{PhC}}^y(\mathbf{r}_D; \omega) = \frac{\omega}{\pi} \frac{1}{\epsilon(\mathbf{r}_D)} \frac{1}{|\tilde{\omega}_\mu|^2} \frac{1}{|a_\mu|^2} \frac{1}{(\omega - \omega_\mu)^2 + \gamma_\mu^2} \times \left\{ \text{Re}(a_\mu) [2\omega_\mu - \omega] \gamma_\mu + \text{Im}(a_\mu) [\omega_\mu (\omega - \omega_\mu) + \gamma_\mu^2] \right\}$$

[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]



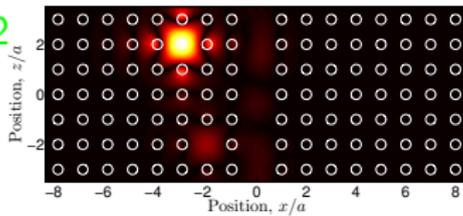
$$\rho_{\text{PhC}}^y(\mathbf{r}_D; \omega) = \frac{\omega}{\pi} \frac{1}{\epsilon(\mathbf{r}_D)} \frac{1}{|\tilde{\omega}_\mu|^2} \frac{1}{|a_\mu|^2} \frac{1}{(\omega - \omega_\mu)^2 + \gamma_\mu^2} \times \left\{ \text{Re}(a_\mu) [2\omega_\mu - \omega] \gamma_\mu + \text{Im}(a_\mu) [\omega_\mu (\omega - \omega_\mu) + \gamma_\mu^2] \right\}$$

[J. R. de Lasson et al., Opt. Lett. **40**, 5790 (2015)]

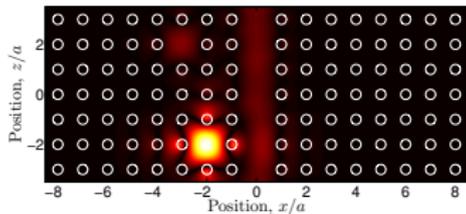


M1

M2

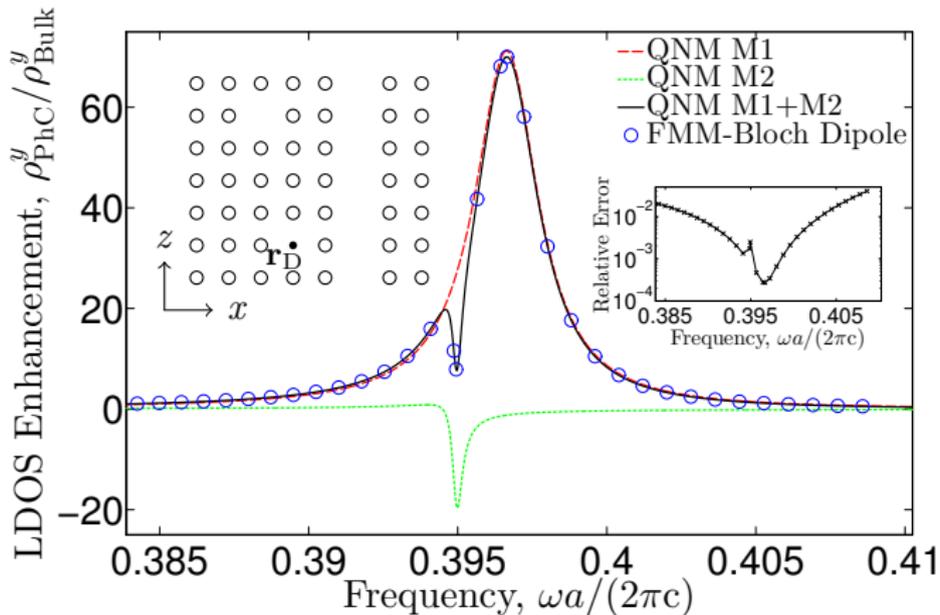
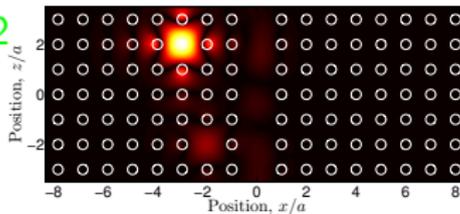


[J. R. de Lasson *et al.*, *Opt. Lett.* **40**, 5790 (2015)]

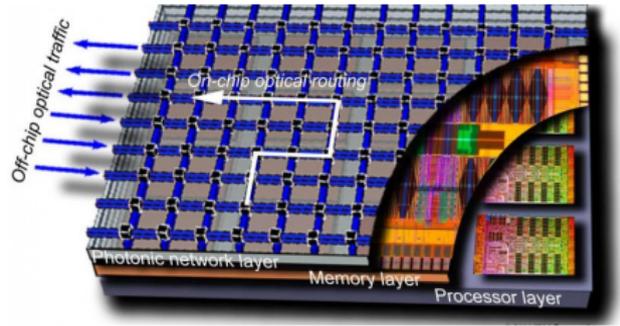


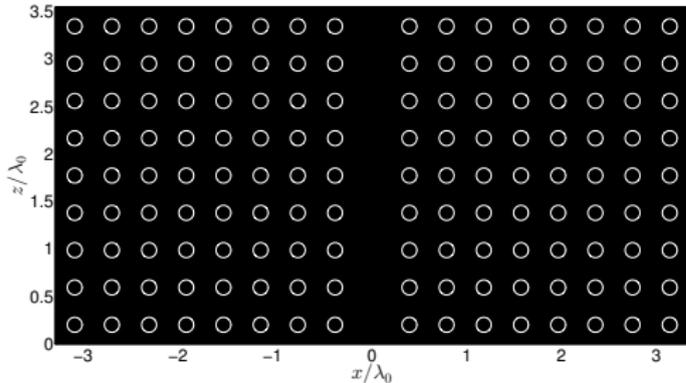
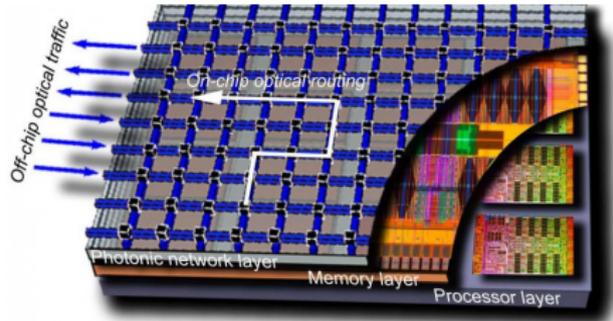
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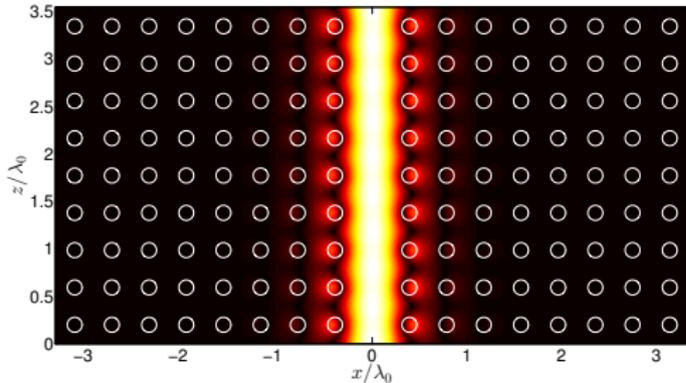
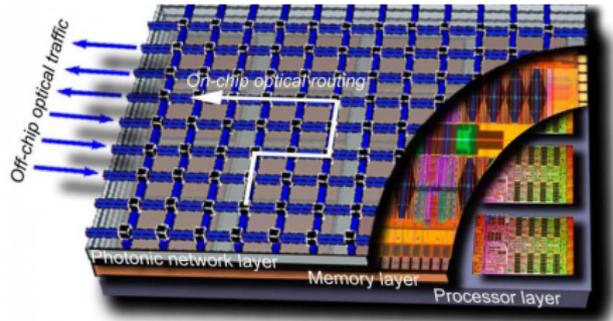
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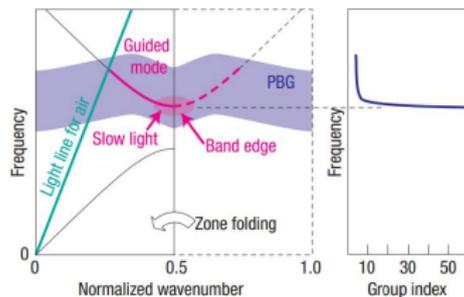
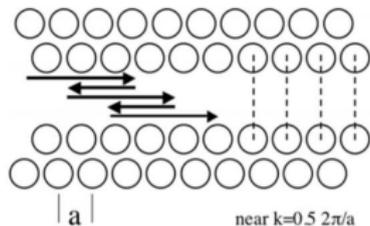
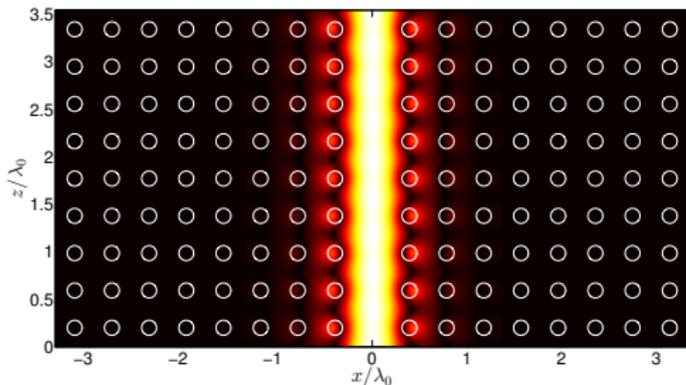
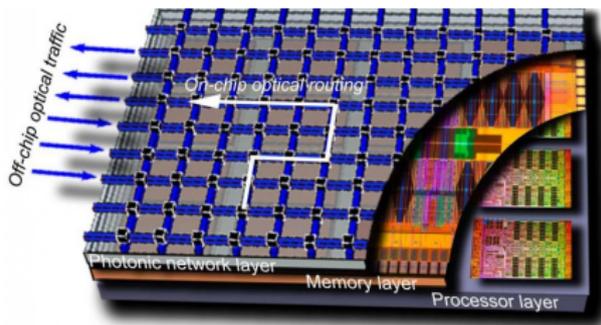


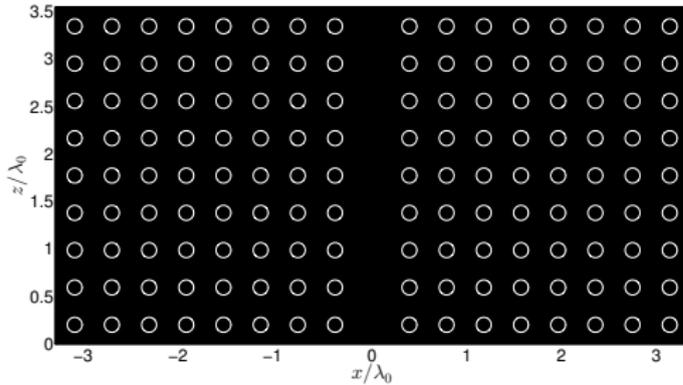
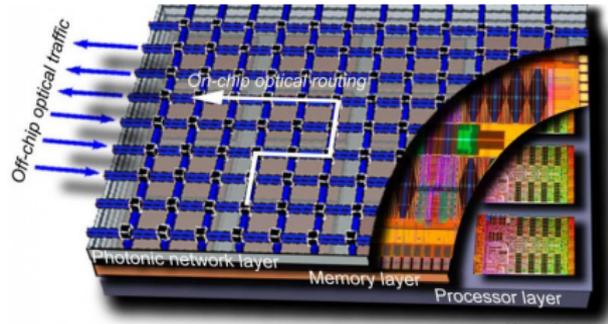
[J. R. de Lasson et al., Opt. Lett. 40, 5790 (2015)]

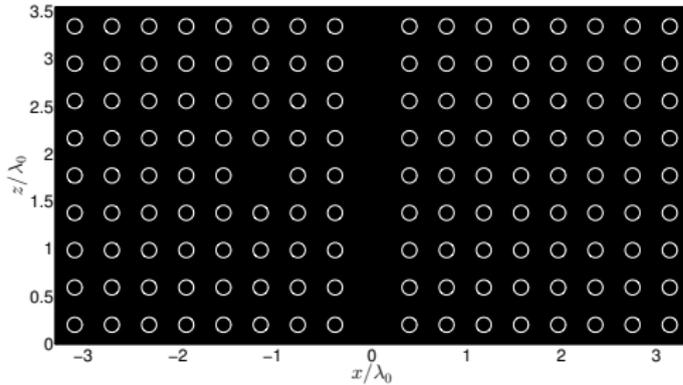
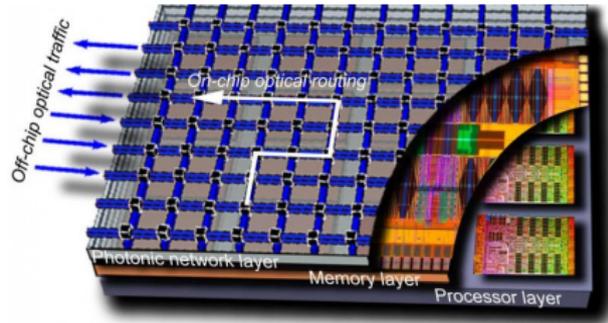


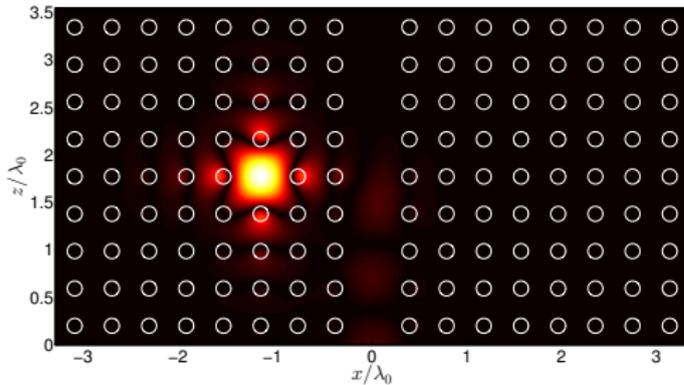
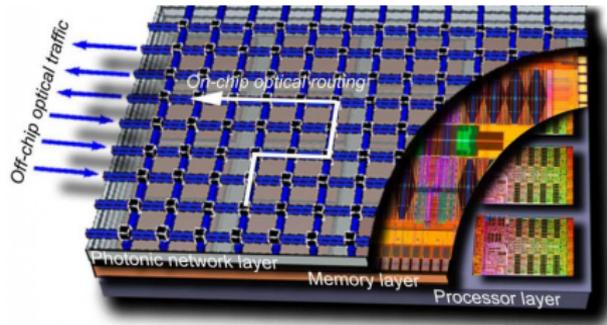


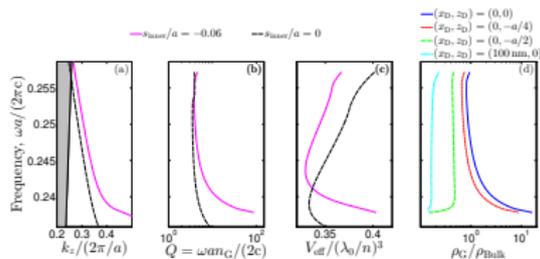
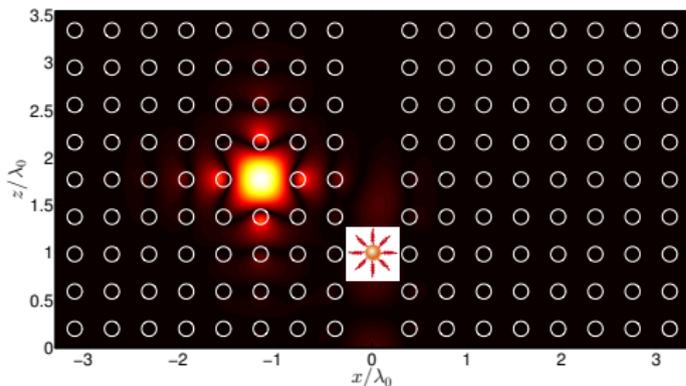
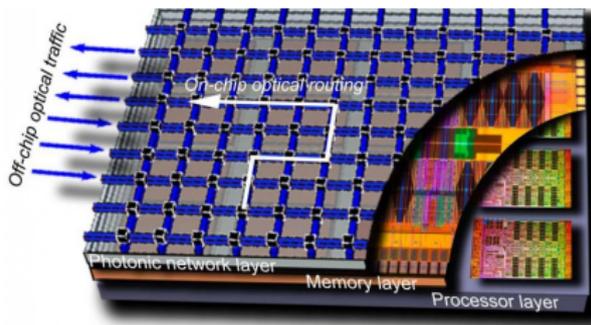


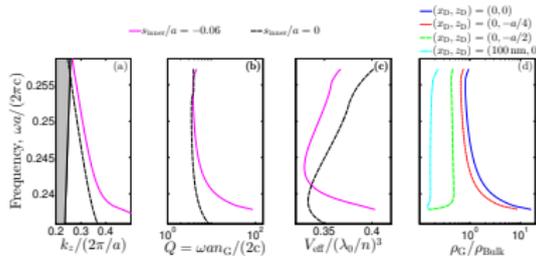
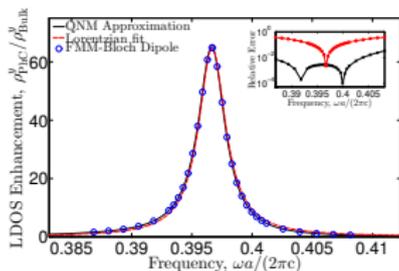
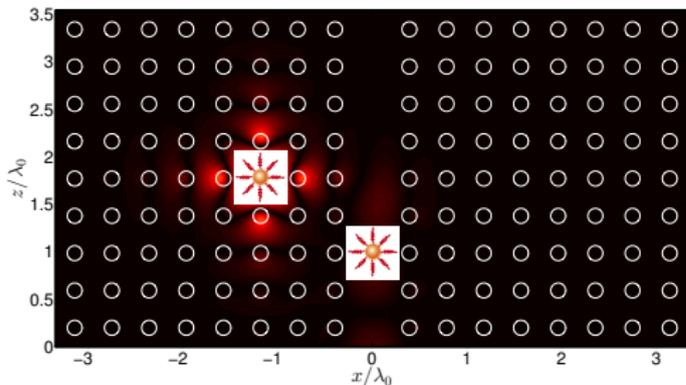
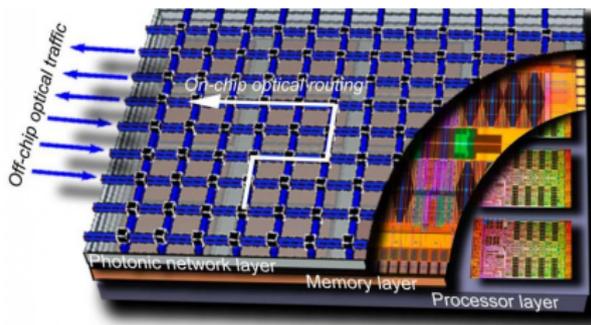


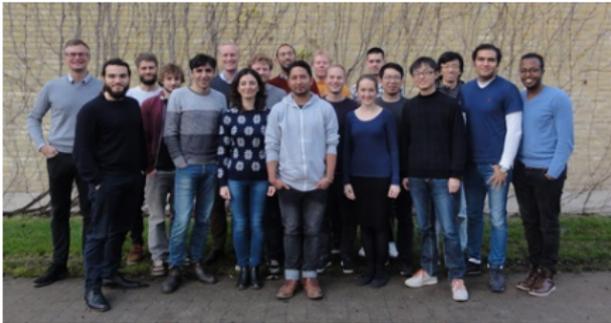




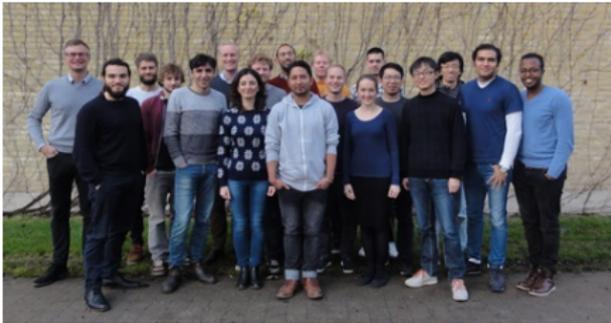








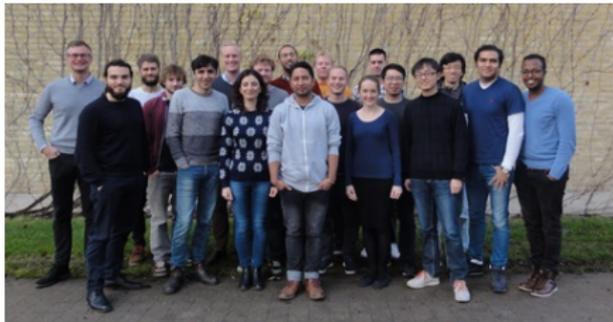
Nanophotonics Theory



Nanophotonics Theory



Nanophotonics



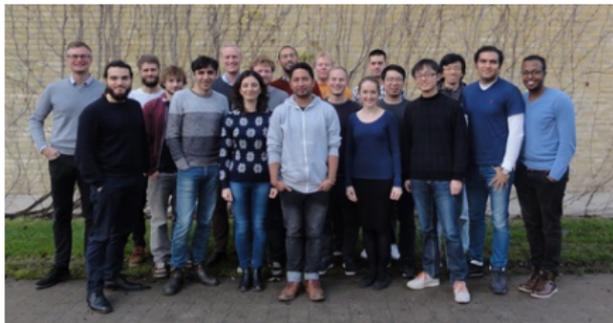
Nanophotonics Theory



Nanophotonics



EPFL-LPN



Nanophotonics Theory



Nanophotonics



EPFL-LPN



Niels



Jesper



Philip



September 3, 2007 → December 18, 2015



September 3, 2007 → December 18, 2015

...Thanks for your attention 😊